Optimizing Low Reynolds Number Locomotion

Anette (Peko) Hosoi
Hatsopoulos Microfluids Laboratory, MIT
What’s in This Talk?

1. Optimal stroke patterns for 3-link swimmers
2. Building a better snail

Swimming

Crawling
Tiny Swimmers

Life at low Reynolds number

E. M. Purcell
Lyman Laboratory, Harvard University, Cambridge, Massachusetts 02138
(Received 12 June 1976)

Editor's note: This is a reprint (slightly edited) of a paper of the same title that appeared in the book Physics and Our World: A Symposium in Honor of Victor F. Weisskopf, published by the American Institute of Physics (1976). The personal tone of the original talk has been preserved in the paper, which was itself a slightly edited transcript of a tape. The figures reproduce transparencies used in the talk. The demonstration involved a tall rectangular transparent vessel of corn syrup, projected by an overhead projector turned on its side. Some essential hand waving could not be reproduced.

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The flexible oar

$-\nabla p + \mu \nabla^2 \mathbf{u} = 0$

$\nabla \cdot \mathbf{u} = 0$

The corkscrew

The 3-link swimmer
Tiny Swimmers

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3-link Swimmer

- Purcell (1977): proposed design
- Becker, Koehler and Stone (2003): optimized geometry (arm length/body length and stroke angle)
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Can we do better?
Optimizing Kinematics
Optimizing Kinematics

Fixed geometry
Optimizing Kinematics

Fixed geometry
Optimizing Kinematics

Fixed geometry

FIG. 2: Stroke sequences of three-link swimmers in the \((\Omega_1, \Omega_2)\)-phase plane for: (−) ... across the axes \(\Omega_1 = \Omega_2\) and \(\Omega_1 = -\Omega_2\). This can be seen by considering a geometrical configuration where \(\Omega_1 = \Omega_2\).
Optimizing Kinematics

Fixed geometry

Kanso and Marsden (2005) - 3-link fish
Berman and Wang (2006) - insect flight
- Lowest order: resistive force theory
- Next order: can incorporate effects of slenderness and interactions between links

Optimized Stroke Patterns

• Two cost functions
  - Efficiency
    - [useful work]/[energy dissipated]
    - Unique parametrization that
      optimizes efficiency for a given curve
  - Speed
• Symmetry axes
Optimized Stroke Patterns

- **Two cost functions**
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3-Link Race

D = distance
W = Total “work”
(viscous dissipation)

Thin line = path of center of mass

Optimize geometry
Optimize geometry AND kinematics

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Multiple Links

- Large $N \rightarrow$ snake
- Analytic solution by Lighthill (in *Mathematical Biofluiddynamics*)
  - 41 degree angle
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  ‣ 41 degree angle
Multiple Links

• Large $N \rightarrow$ snake
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  ‣ 41 degree angle

small drag coefficient

large drag coefficient
Effect of Slenderness

from Becker et al.

Fully optimized stroke
'Purcell' stroke

Efficiency $\mathcal{E}$

Slenderness $\log \frac{1}{\kappa}$
Effect of Slenderness

from Becker et al.

Biological systems

Fully optimized stroke

'Purcell' stroke

Efficiency $\mathcal{E}$ vs. Slenderness $\log \frac{1}{\kappa}$
Gastropod Locomotion

Locomotion is directly coupled to stresses in the thin fluid film
Gastropod Locomotion

Locomotion is directly coupled to stresses in the thin fluid film
Gastropod Locomotion

Locomotion is directly coupled to stresses in the thin fluid film.

Retrograde vs direct waves

What is Required for Locomotion?
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$F_{\text{shear stress}}$
What is Required for Locomotion?

\[ F = \tau_1 A = \mu(\tau_1) V_1 A / H \]

\[ V_{cm} = \frac{[V_2(N - 1) + V_1]}{N} \]

Couette flow in small gap

Each pad carries equal mass

F shear stress
What is Required for Locomotion?

\[ F = \tau_1 A = \mu(\tau_1) V_1 A/H \]

\[ V_{cm} = [V_2(N - 1) + V_1] / N \]

\[ V_{cm} = \frac{FH}{AN} \left( \frac{1}{\mu(\tau_1)} - \frac{1}{\mu(\tau_2)} \right) \]

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\( F \) shear stress
What is Required for Locomotion?

Couette flow in small gap

Each pad carries equal mass

Nonlinear characteristics first measured by:

$F = \tau_1 A = \mu(\tau_1)V_1 A/H$

$V_{cm} = \frac{[V_2(N-1) + V_1]}{N}$

$V_{cm} = \frac{FH}{AN} \left( \frac{1}{\mu(\tau_1)} - \frac{1}{\mu(\tau_2)} \right)$

$V_{cm} = \frac{[V_2(N-1) + V_1]}{N}$
RoboSnail II

[Image of a mechanical setup and graphs showing rheological properties of Laponite and Carbopol solutions]
“Tune” Material Properties

?
“Tune” Material Properties

![Graph showing stress vs. strain rate with three types of material behavior: Newtonian, Shear Thickening, and Shear Thinning. The graph has a question mark in the center, indicating an area of uncertainty or inquiry.](image-url)
“Tune” Material Properties

- Perturb rheology of Newtonian fluid (snail will crawl on any non-Newtonian fluid)
  - shear thickening
  - shear thinning
- Which material properties are “favorable”??
“Tune” Material Properties

- Perturb rheology of Newtonian fluid (snail will crawl on any non-Newtonian fluid)
  \[
  \dot{\gamma} = \frac{\sigma}{\mu} \left( 1 - \epsilon \frac{|\sigma|}{\sigma_*} \right)
  \]
  - \( \epsilon > 0 \) shear thickening
  - \( \epsilon < 0 \) shear thinning
- Which material properties are “favorable”?
• **Mechanical work** done in crawling ( = rate of viscous dissipation)

\[
\mathcal{E} = \int_0^\lambda \int_0^h \sigma \dot{\gamma} \, dy \, dx.
\]

\[
\mathcal{E} = \frac{h \lambda}{4 \mu} \langle \sigma^2 \rangle - \epsilon \frac{17}{96} \frac{h \lambda}{\mu \sigma_*} \langle \sigma^2 | \sigma | \rangle + \ldots
\]
Rheology Cost Function

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> 0
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\(\epsilon > 0\)  shear thickening

\(> 0\)

\(\ldots\)
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\[ \epsilon > 0 \quad \text{shear thickening} \]

• Chemical cost associated with mucus production ( ~ flux in frame moving with snail)

\[ Q = \int_0^h u_s(x) \, dy \quad \rightarrow \quad Q_s = \epsilon \frac{79}{432} \frac{h^2}{\mu \sigma^*} \langle \sigma | \sigma | \rangle \left( 1 + \epsilon \frac{185}{8532\sigma^*} \frac{\langle \sigma^3 \rangle - \langle |\sigma| \rangle \langle \sigma | \sigma | \rangle}{\langle \sigma | \sigma | \rangle} + \ldots \right) \]
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  \( \epsilon < 0 \) shear thinning
“The high cost is primarily due to the cost of mucus production, which alone is greater than the total cost of movement for a mammal or reptile of similar weight, ...”

shear thinning
Cost of Locomotion

“The high cost is primarily due to the cost of mucus production, which alone is greater than the total cost of movement for a mammal or reptile of similar weight, ...”

Mark Denny, Science, 208, No. 4449 (1980)

Pedal mucus from common garden snail, Helix aspera is strongly shear-thinning.
Final Comments

- 3-link (and n-link) swimmer (low Reynolds number)
  - Optimizing kinematics
  - Trade-off between efficiency and robustness in biological systems?

- Snails
  - Rely on the nonlinear response of pedal mucus to crawl
  - We can “tune” viscous material properties to find which weakly nonlinear response is energetically favorable → shear thinning
  - Mechanical wall-climber
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- Optimizing 3-link swimmer
- Robosnails + mechanical swimmer
- Slip, Swim, Mix, Pack: Fluid Mechanics at the Micron Scale

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