The Planck constant, $h$, and the redefinition of the SI

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National Institute of Standards and Technology (NIST)

MASPG
Mid-Atlantic Senior Physicists Group
09-19-2012
Outline

1. The SI and the definition of the kg

2. The principle of the Watt balance

3. The past, the present, & the future of the Ekg
A brief history of units

I. Based on man.

II. Based on Earth.

III. Based on fundamental constants.
The current SI†

† Système international d’unités
The current SI†

†Système international d’unités
The new SI†

†Système international d’unités
Why fix it, if it is not broken?
The kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (1889)
The kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (1889)

CIPM 1989: The reference mass of the international prototype is that immediately after cleaning and washing by a specified method.
Cylindrical
height: 39 mm, diameter: 39 mm
Alloy 90% platinum and 10% iridium

International Prototype: IPK
Official Copies: K1, 7, 8(41), 32, 43, 47
BIPM prototypes
  for special use: 25
  for routine use: 9, 31, 67
National prototypes:
  12, 21, 5, 2, 16, 36, 6, 20, 23, 37,
  18, 46, 35, 38, 24, 57, 39,
  40, 50, 48, 44, 55, 49, 53
  56, 51, 54, 58, 68, 60, 70,
  65, 69
New prototypes 67, 71, 72, 74, 75, 77-94
Other prototypes 34

USA:
  K4 (1890) check standard
  K20 (1890) national prototype
  K79 (1996)
  K85 (2003) watt balance
  K92 (2008)

1889: -39 μg
1950: -19 μg
1990: -21 μg
Let’s look at washing

<table>
<thead>
<tr>
<th>Prototype</th>
<th>date of previous cleaning and washing</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>22 October 1982</td>
</tr>
<tr>
<td>K1</td>
<td>Sept / Oct 1957</td>
</tr>
<tr>
<td>8(41)</td>
<td>12 March 1965</td>
</tr>
<tr>
<td>43</td>
<td>Sept / Oct 1957</td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>14 September 1946</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

The cleaning procedure takes 1 \( \mu g \)/year for each year since the last cleaning procedure occurred.
How stable is the kilogram?

The mass of 6 out of 7 has increased.
Shortcomings

- Can be damaged or destroyed.
- Collects dirt from the ambient atmosphere.
- Cannot be used routinely for fear of wear.
- IPK changes by 50µg/100yrs relative to the ensemble of PtIr standards.
- The drift of the world wide ensembles of PtIr standards is unknown at a level of 1mg/100yrs.
- Can only be accessed at the BIPM.
- Cannot be communicated to extraterrestrial intelligence.
The kilogram influences other units

- derived unit
- base units

\[
1 \, V = 1 \, \frac{J}{As} = 1 \, \frac{kg \, m^2}{As^3}
\]
Two advances in metrology in “recent years”

1. Josephson Effect

   Josephson Junction (JJ) array

2. Integral Quantum Hall Effect

   Quantum Hall Resistor
Josephson Junction

- Weak link between two superconductors

\[ \Psi_1 = A_1 e^{\theta_1}, \quad \Psi_2 = A_2 e^{\theta_2} \]

\[ \phi = \theta_2 - \theta_1 \]

- Electrical properties

\[ V = \frac{h}{2e} \frac{1}{2\pi} \frac{d\phi}{dt} \]

or,

\[ V = \frac{h}{2e} \int f \]

DC only

with AC
JJ array

20208 Junctions
-14 to +14 Volts
typical: f=14..75 GHz

Voltage

19 mm
Quantum Hall Effect

\[ R_H = \frac{V_H}{I} = \frac{B}{eN_s} \]

\[ N_s = \left( \frac{eB}{h} \right)i \]

\[ R_H = \left( \frac{h}{ie^2} \right) \]

\[ T = 278 \text{ mK} \]
\[ I = 0.255 \text{ } \mu\text{A} \]
Two new constants

- Josephson constant

\[ K_J = \frac{2e}{h} = (483597.870 \pm 0.011) \frac{\text{GHz}}{V} \]

- von Klitzing constant

\[ R_K = \frac{h}{e^2} = (25812.8074434 \pm 0.0000084) \Omega \]

- \[ K_{J-90} = 483597.9 \frac{\text{GHz}}{V} \]

- \[ R_{K-90} = 25812.807 \Omega \]
A schism in metrology

SI units
(base and derived)

conventional units
Difference between SI and 90 units

\[
\begin{align*}
1 \text{ V} &= (1 - 62.0 \times 10^{-9} \pm 22.7 \times 10^{-9}) \text{ V}_{90} \\
1 \text{ \Omega} &= (1 - 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9}) \text{ \Omega}_{90} \\
1 \text{ A} &= (1 - 44.9 \times 10^{-9} \pm 22.7 \times 10^{-9}) \text{ A}_{90} \\
1 \text{ C} &= (1 - 44.9 \times 10^{-9} \pm 22.7 \times 10^{-9}) \text{ C}_{90} \\
1 \text{ W} &= (1 - 106.9 \times 10^{-9} \pm 45.5 \times 10^{-9}) \text{ W}_{90} \\
1 \text{ F} &= (1 + 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9}) \text{ F}_{90} \\
1 \text{ H} &= (1 - 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9}) \text{ H}_{90}
\end{align*}
\]

Redefinition by committee

Oct 2011 vote on Draft resolution

passed

4 year wait

Ready to redefine

yes

new SI

no
# The current SI

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second</td>
</tr>
<tr>
<td>kg</td>
<td>The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.</td>
</tr>
<tr>
<td>s</td>
<td>The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.</td>
</tr>
<tr>
<td>A</td>
<td>The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length.</td>
</tr>
<tr>
<td>K</td>
<td>The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.</td>
</tr>
<tr>
<td>mol</td>
<td>The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is “mol.”</td>
</tr>
<tr>
<td>cd</td>
<td>The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian.</td>
</tr>
</tbody>
</table>
The “new” SI

Based on seven reference constants.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \nu^{(133\text{Cs})}$</td>
<td>ground state hfs</td>
<td>9.192 631 770</td>
<td>s$^{-1}$</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
<td>2.99 792 458</td>
<td>ms$^{-1}$</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant</td>
<td>6.626 069 57*</td>
<td>kg m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$e$</td>
<td>elementary charge</td>
<td>1.602 176 565</td>
<td>As</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann constant</td>
<td>1.380 6448</td>
<td>kg m$^2$ s$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>$N_A$</td>
<td>Avogadro constant</td>
<td>6.022 141 29</td>
<td>mol$^{-1}$</td>
</tr>
<tr>
<td>$K_{cd}$</td>
<td>luminous efficacy</td>
<td>6.83</td>
<td>lm kg$^{-1}$ m$^{-2}$ s$^{3}$</td>
</tr>
</tbody>
</table>

* The exact numerical value will be determined by CODATA at the time of the redefinition. The above values are today’s CODATA values.
Redefinitions do happen

Adjustments of national voltage standards

\[ K_{J-90} = 483597.9 \frac{\text{GHz}}{\text{V}} \]

- USSR 1990: 4.5 ppm
- USSR 1972: 3.565 ppm
- France 1990: 6.741 ppm
- France 1972: 1.32 ppm
- US 1990: 9.264 ppm
- US 1972: -1.2 ppm
- Other countries 1990: -8.065 ppm

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2. The principle of the Watt balance

3. The past, the present, & the future of the Ekg
The principle of the Watt balance

**force mode**

\[ mg = -NevB = IlB \]

\[ lB = \frac{mg}{I} \]

**velocity mode**

\[ evB = eE = e \frac{V}{l} \]

\[ lB = \frac{V}{v} \]
The Watt equation

\[ mg \ v = V \ I \]
The Watt equation

\[ mgv = VI \]

Virtual power!
Connection to Planck’s constant

velocity mode

\[ V = c_V f_V \frac{h}{2e} \]

\[ \frac{h}{2e} = 2.066.. \frac{\mu V}{GHz} \]
Connection to Planck’s constant

force mode

R is calibrated against a Quantum Hall Resistor $R_K$

$$R = c_R R_K = c_R \frac{h}{e^2}$$

$$I = \frac{V_R}{R} = \frac{c_{RV} f_R}{2e} \frac{h}{c_R \frac{h}{e^2}}$$
Combining the two modes

\[ V I = V \frac{V_R}{R} = \frac{c_V f_V h}{2e} \frac{c_{RV} f_R}{2e} = \frac{c_V c_{RV}}{c_R^4} f_R f_V h \]

\[ m = \frac{c_V c_{RV}}{4c_R} \frac{f_v f_{VR}}{g_V} h \]

Mass can be defined in terms of Planck’s constant.
Gravity

Absolute gravimeter, can measure $g$ to 1 ppb

Feb 6 - Feb 10 2012
Worldwide Watt Balances (WWB)

BIPM (France)

METAS (Switzerland)

LNE (France)

NIM (China)
MSL (New Zealand)
KRISS (South Korea)
PTB (Germany)

NPL (UK)/NRC (Canada)
A digression... ...

- There is another way to measure $h$
- The International Avogadro Project

PTB, Germany
NMIJ, Japan
NMI, Australia
METAS, Switzerland
NIST, US
INRIM, Italy
BIPM
IRMM, Belgium
Si sphere (Avogadro project)

A single crystal of Si

\[ \rho = \frac{m}{V} \]

\[ V_{mol} = \frac{M_{mol}}{\rho} \]

Unit cell (8 atoms per cell)

\[ V_{uc} = a_0^3 = 8^{3/2} d_{220}^3 \]

\[ N_A = 8 \frac{V_{mol}}{V_{uc}} = 8 \frac{M_{mol} V}{m 8^{3/2} d_{220}^3} \]
Measurements needed

\[ N_A = 8 \frac{V_{\text{mol}}}{V_{uc}} = 8 \frac{M_{\text{mol}} V}{m 8^{3/2} d_{220}^3} \]

- physics and chemistry of bulk and surface
- isotopic abundance & mass spectroscopy
- volume determination
- mass metrology
- XRD
Measurements needed

\[ N_A = 8 \frac{V_{mol}}{V_{uc}} = 8 \frac{M_{mol} V}{m 8^{3/2} d_{220}^3} \]

Physics and chemistry of bulk and surface

Isotopic abundance & mass spectroscopy

Volume determination

Mass metrology

Measurement completed in 2003

Largest contribution to the uncertainty: measurement of the isotopic abundance

Solution: Use enriched Si.
Pictures

5kg $^{28}$Si crystal grown by the float zoning method
Connection between $h$ and $N_A$

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$
Connection between $h$ and $N_A$
Connection between $h$ and $N_A$

\[ R_\infty = \frac{m_e c \alpha^2}{2h} = \frac{m_e c \alpha^2 M_p / N_A}{2hm_p} = \frac{M_p c \alpha^2}{2h(m_p / m_e)N_A} \]
Connection between $h$ and $N_A$

\[
R_\infty = \frac{M_p c \alpha^2}{2h(m_p / m_e) N_A}
\]

- $5 \times 10^{-12}$
- $8.9 \times 10^{-11}$ (fixed)
- $3.2 \times 10^{-10}$
- $4.1 \times 10^{-10}$
Outline

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The birth of the Watt balance

A MEASUREMENT OF THEGYROMAGNETIC RATIO OF THE
PROTON BY THE STRONG FIELD METHOD

B. P. Kibble
Division of Electrical Science
National Physical Laboratory, Teddington, England

4 A SUGGESTION FOR A DIFFERENT WAY OF REALISING THE AMPERE

A major aim of these measurements is to determine the ratio of the maintained ampere to the SI ampere, denoted by K, by combining the result with that of the weak field method (Cohen and Taylor 1973). We take this opportunity to draw attention to a possible way of determining K directly which needs only minor modifications to the strong field apparatus described above.
NPL-1

One shortcoming

velocity mode (wire cold)

\[ \frac{V}{v} = Nl_1B \]

change in temperature

\[ l_2 = l_1(1 + \alpha \Delta T) \]

change in l

weighing mode

\[ \frac{mg}{I} = l_2B \]

Copper: \( \alpha = 16.6 \times 10^{-6} \text{ K}^{-1} \)

I=10 mA, R=100 \( \Omega \)
The solution (idea by P.T. Olsen)

cold coil

\[ B(r) = \frac{C}{r} \]

warm coil

\[ B(r) = \frac{C}{r} \]

\[ \frac{V}{v} = N 2\pi r_1 \frac{C}{r_1} = N 2\pi C \]

\[ \frac{mg}{I} = N 2\pi r_2 \frac{C}{r_2} = N 2\pi C \]
The NIST way

Solenoid current $cw$

$2r$

$d$

$h$

Solenoid current $ccw$

region, where $B(r)=C/r$

---

NIST-1, conventional EM

3 kW, 8A, 0.003T

superconducting EM

5A, 0.1T

balance in air

NIST-2

NIST-3

rel. uncertainty ($10^{-9}$)

10000

1000

100

10


year

Physical Measurement Laboratory
NIST-3 Apparatus
The weighing mode (in more detail)
The weighing mode (in more detail)

\[ I_- N 2\pi C = mg - I_+ N 2\pi C \]

\[ mg = (I_- + I_+) N 2\pi C \]

Two advantages:

1.) Substitution method
2.) Current is halved, Heating is quartered
The velocity mode (in more detail)
9 cm
5 cm
4 cm
Band of 70 Pt-W wires

Physical Measurement Lab

1 cm
Voltage velocity measurement

![Graph showing voltage velocity measurement with position in cm on the x-axis and ratio (V/s/m) on the y-axis. The graph displays a wave-like pattern with tick marks at 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 cm.](image)

- The vertical scale on the left is labeled as (mV/s/m) ranging from 491.880 to 491.970.
- The horizontal scale on the bottom is labeled as Position (cm) ranging from 1 to 10.
- The graph includes a blue line with a purple vertical bar indicating a sensitivity of $10^{-4}$, and another purple vertical bar indicating a sensitivity of $10^{-5}$.
NPL-2 = NRC-1

- equal arm beam balance
- design with 3 knives
- SmCo permanent magnet
- radial field
NRC-1 magnet design
The present!

\[ \frac{(h-h_{\text{CODATA}})}{h_{\text{CODATA}}} \text{ (ppb)} \]

-500  0  500  1000  1500

NRC-1, 2012
NRC, \(^{28}\text{Si}, 2012\)
CODATA, 2010
NPL-2, 2012
METAS-1, 2011
Avogadro, \(^{28}\text{Si}, 2010\)
NIST-3, 2007
NPL-2, 2007
NIST-3, 2005
Avogadro, \(^{\text{nat}}\text{Si}, 2003\)
NIST-2, 1998
NPL-1, 1990

\[ h \left(10^{-34} \text{ Js}\right) \]
The present!

We will figure this out!

Correlated:
- NRC-1, 2012
- NRC, $^{28}$Si, 2012
- CODATA, 2010

Discrepant:
- NPL-2, 2012
- METAS-1, 2011
- Avogadro, $^{28}$Si, 2010
- NIST-3, 2007
- NPL-2, 2007
- NIST-3, 2005
- Avogadro, $^{nat}$Si, 2003
- NIST-2, 1998
- NPL-1, 1990

$h$ (10$^{-34}$ Js)

(h-h$_{CODATA}$)/h$_{CODATA}$ (ppb)
NIST-3 v2.0

- A new, (mostly independent) team will measure a last data point with NIST-3.

- Blind measurement: The mass group will measure our masses. But they will add a to us unknown offset (-500ppb...500ppb) to the values.
RECOMMENDATIONS OF THE
CONSULTATIVE COMMITTEE FOR MASS AND RELATED QUANTITIES
SUBMITTED TO THE INTERNATIONAL COMMITTEE FOR WEIGHTS AND MEASURES

RECOMMENDATION G 1 (2010)
Considerations on a new definition of the kilogram

The Consultative Committee for Mass and Related Quantities (CCM)

*recalling* its previous Recommendation to the CIPM on the “Conditions for a new definition of the kilogram”, CCM G 1 (2005), and

*considering*
recommends

- that the following conditions be met before the kilogram is redefined in terms of fundamental constants:

  1. at least three independent experiments, including work both from watt balance and from International Avogadro Coordination projects, yield values of the relevant constants with relative standard uncertainties not larger than \(5 \text{ parts in } 10^8\). At least one of these results should have a relative standard uncertainty not larger than \(2 \text{ parts in } 10^8\),

  2. for each of the relevant constants, values provided by the different experiments be consistent at the 95 % level of confidence,

  3. traceability of BIPM prototypes to the international prototype of the kilogram be confirmed,

- that the CODATA recommended values be adopted for the relevant fundamental constants,

- that the associated CODATA relative standard uncertainties be suitably considered when the initial uncertainty is assigned to the mass of the international prototype of the kilogram.
Timeline

01 Oct 2011
now, 09/01/12

Solenoid cooled down

Assessment Planning Order of hardware

new team starts working on NIST-3

Change of electrical grounding
New PJVS

Comparison of absolute gravimeter

01 Oct 2013

Goal: An independent data point with NIST-3
Independent measurement & uncertainty analysis

Alignment

First quarter of data uncertainties = 100 x 10^-9

New
• current source
• vacuum pumps
• knife edge
Upgrades to
• Interferometers

Comparison of JJ Voltage Std.

Data analysis, systematic tests

Physical Measurement Laboratory
Changes

Power Filters

GND STAR

PJVS

Autocollimator for laser alignment
Changes continued

- Stabilization of SC current
- Zener
- Standard Cell
- Current source
- Laser for velo meas
- In situ laser calibration
- $I_2$ stabilized HeNe
Results

Bad connection to current source

JJA grounded

many experiments

1 kg
0.5 kg
In the meantime we are building NIST-4

NIST-3

“The Thoroughbred is a horse breed best known for its use in horse racing. Thoroughbreds are considered “hot-blooded” horses, known for their agility, speed and spirit.”

• Watt balance to measure $h$
• Optimized for the best measurement of $h$
• Highly sophisticated
• Costly to operate and maintain
• Lifetime: $\approx 10$ years

NIST-4

“The Clydesdale is a cold blooded horse appreciated for its strength, style, and versatility. Like all cold-blooded horses the Clydesdale has a stolid demeanor and is not suitable for sports other than hauling or pulling.”

• Watt balance to realize the kg
• Optimized for reliable operation
• Ease of Use
• Low maintenance
• Lifetime: $>30$ years
NIST-4

• Design has started.
• Baseline design: Wheel balance with permanent magnet system.
• Magnet is designed and plans are at a manufacturer.
• Prepare the infrastructure.
• Think about alternative design ideas.
NIST-4 magnet

- SmCo magnet rings provide field.
- Wide gap (3cm) 15 cm long.
- Symmetric design, field minimum.
- Field $\approx 0.54$ T.
- Magnet separates.
- Full iron enclosure provides shielding.

8 parts:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>outer yoke</td>
</tr>
<tr>
<td>5-7</td>
<td>inner yoke</td>
</tr>
<tr>
<td>4 &amp; 8</td>
<td>magnets</td>
</tr>
</tbody>
</table>

Construction is about to start!
Access to the coil

To access the coil, the magnet separates:

Total mass of the magnetic circuit: 850 kg
Mass of one magnet ring: 44 kg
Mass of the lower part of the magnet: 330 kg
**FEA results**

- Calculated by MagNet
- Flux in Iron < 0.9 T
- Field in gap is 0.54 T
- Coil radius is 21.5 cm

\[ F = N2\pi rBI \]

\[ \frac{F}{I} = N0.732 \frac{N}{A} \]

\[ \frac{F}{I} = 500..1000 \frac{N}{A} \]

\[ N = 680..1370 \]
Field quality

The graph shows the horizontal B-field (T) as a function of the vertical position, z (cm). The main graph displays the general trend, while the zoomed view highlights the B(0) value of 0.542 ppm.
Vacuum Vessel is designed
Magnet in vacuum vessel
NIST-4 next steps

- Prepare infrastructure (crane, clean power, vacuum pumps).
- Building a device to verify the magnet.
- Work on an alternative mechanical design.
- Design load lock for the masses.
Outline

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3. The past, the present, & the future of the Ekg
Thank you for your attention!

Jon Pratt, group leader

part-time:

Stephan Schlamminger  Darine Haddad  Frank Seifert  Ruimin Liu  David Newell

Leon Chao  Ed Williams  Shawn Zhang
The End
Summary

• Current definition of the kilogram.
• Shortcomings of the definition.
• Quantum electrical metrology.
• Principle of the Watt balance (velocity mode, weighing mode).
• Account of the past, present and future of the electronic kilogram
• Introduction to the NIST-4 Watt balance
• Top 2/3 of the magnet.
• The lower 1/3 is a meter below.
• At the beginning we can use the top 2/3 of the magnet to tweak our system. We should be able to make a complete Watt balance experiment with that, until we have settled for an induction coil.
• The field is not as uniform.
• Shielding is not perfect.
Field quality of the upper 2/3

![Graph showing the horizontal B-field (T) against the vertical position, z (cm). The graph includes a zoomed view showing B(0) = 0.354.](image-url)
External Demagnetizing field on PM

recoil curve: $B(H) = B_r + \mu_0 \mu_r H$

load curve 1: $B(H) = -\mu_y mH$

load curve 2: $B(H) = -\mu_0 mH - \mu_0 mnI$

working point 1: $H_{wp1} = \frac{-B_r}{\mu_0 (\mu_r + m)}$

working point 2: $H_{wp2} = \frac{-B_r}{\mu_0 (\mu_r + m)} - \frac{mnI}{(\mu_r + m)}$

change in $B$: $\Delta B = \frac{m \mu_0 \mu_r nI}{\mu_r + m}$

smaller $m$ is better
Additional H-field at the Magnet

Result from FEA:

- Top magnet: \( H_{\text{ext}} \) is parallel to \( B \)
- Bottom magnet: \( H_{\text{ext}} \) is anti-parallel to \( B \)

Since there are two magnets, the effect cancels. One magnet gets stronger, the other weaker by the same amount!
Semi Analytic result

\[ \text{slope: } 1.09 \mu_0 \]

\[ 70 \text{ kA/m (1000x than with weighing current)} \]

\[ \text{slope: } -1.75 \mu_0 \]

\[ 0.06 \text{ T (10\%)} \]

\[ \text{25}^{\circ} \text{C} \]

unload magnet
upper magnet
lower magnet

B (solid) or \( \mu_0 M \) (dashed) (T)

H (kA/m)
Yoke Material at working point
H-field in Yoke due to weighing current

H in Yoke < 0.2 A/m

0.2/800 = 250 ppm

6.82 A·t (10 mA 682 turns)
H-field in PM caused by weighing current

H in Magnet ≈ 70 A/m, see next slide.

6.82 A-t (10 mA 682 turns)
Uncertainty as a function of mass

![Graph showing the relationship between uncertainty and mass on a log-log scale. The curve indicates a decrease in uncertainty with increasing mass, reaching a minimum at a certain mass value and then increasing again. A red line labeled "heating effect (very conservative)" intersects the curve at a low mass value, suggesting a conservative approach to estimating uncertainty.](image-url)
The Watt balance master equation

\[
\frac{h}{h_{90}} = \frac{W_{90}}{W} = \frac{\{mgv\}_{SI}}{\{UI\}_{90}}
\]

with

\[
h_{90} = \frac{4}{K_{J-90}^2 R_{K-90}} = 6.62606885436 \times 10^{-34} \text{ Js}
\]