MORPHODYNAMICS OF RIVERS AND TURBIDITY CURRENTS:
AN ELEGANT CONVERSATION BETWEEN WATER AND SEDIMENT

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A CIVIL ENGINEER/GEOLOGIST GIVING AN INVITED TALK AT THE AMERICAN PHYSICAL SOCIETY

IS LIKE A COUNTRY PRIEST GRANTED AN AUDIENCE WITH THE POPE
“PURE FLUID MECHANICS”

Haboob dust storm

Capillary waves

Roll waves

Wind ripples

Image courtesy IOCC

http://www.rikenresearch.riken.jp/research/223/images/2234070426115631.jpg
http://scribalterror.blogs.com/scribal_terror/images/2007/05/02/dust_2.jpg
http://images.jupiterimages.com/common/detail/66/41/23354166.jpg
HYDRAULIC JUMPS AND BORES

Jump in mountain river

Tidal bore

Circular jump in kitchen sink

Atmospheric hydraulic jump

http://pasternack.ucdavis.edu/falls/aircontent/images/firstthreat.jpg
http://imgi.uibk.ac.at/mmetgroup/trex/webstyle/sierrawave.png
NON-SEDIMENT FLUID-BOUNDARY INTERACTION: MEANDERING CHANNELS IN ICE

http://people.whitman.edu/~carsonrj/researchpics/MendenhallAK2.jpg
THE EFFECT OF INCREASING THE WIDTH-DEPTH RATIO B/H

Flume with flow off

Tributary of Amazon River

Image courtesy H. Ikeda

Dunes

Courtesy National Geographic
Flume with flow off \( B/H \rightarrow UP \) Rhine River, Switzerland

Single-row alternate bars

Courtesy H. Ikeda

Courtesy M. Jaeggi
Flume with flow off  \( \text{B/H} \rightarrow \text{UP} \)  Fuefuki River, Japan

Multiple-row alternate bars

Courtesy H. Ikeda  
Courtesy S. Ikeda
Flume with flow off B/H → UP Ohau River, New Zealand

Braiding

Courtesy H. Ikeda
RIVER DUNES

Fly River, Papua New Guinea

Confluence of Parana and Bermejo River, Argentina

Courtesy M. Amsler, J. Best, D. Parsons etc.
DUNES IN THE RHINE DELTA, THE NETHERLANDS

Image courtesy A. Wilbers and A. Blom
DUNE ASYMMETRY

Dunes in a channel at St. Anthony Falls Laboratory, University of Minnesota, USA

Dunes in a channel at Tsukuba University, Japan. Image courtesy H. Ikeda.
The parameters:

- \( x \) = streamwise distance \([L]\)
- \( t \) = time \([T]\)
- \( \eta \) = bed elevation \([L]\)
- \( q_t \) = volume total sediment transport rate per unit stream width \([L^2/T]\)
- \( \zeta = H + \eta \) = water surface elevation \([L]\)
- \( \lambda_p \) = bed porosity \([1]\)
- \( g \) = acceleration of gravity \([L/T^2]\)
- \( H \) = flow depth \([L]\)
- \( U \) = depth-averaged flow velocity \([L/T]\)
- \( q_w = UH \) = water discharge per unit stream width \([L^2/T]\)
OCCAM’S RAZOR: THE MIMINAL FORMULATION TO ANSWER THE QUESTION

Shallow-water inviscid equations of mass and momentum balance

\[
\frac{\partial H}{\partial t} + \frac{\partial UH}{\partial x} = 0 \quad \rightarrow \quad UH = q_w = \text{const} \tan t
\]

\[
\frac{\partial UH}{\partial t} + \frac{\partial U^2H}{\partial x} = - \frac{1}{2} gH \frac{\partial H}{\partial x} - \frac{1}{2} gH \frac{\partial \eta}{\partial x}
\]

Quasi-steady assumption: \( q_t/q_w << 1 \)

Exner’s equation of conservation of bed sediment:

\[
(1 - \lambda_p) \frac{\partial \eta}{\partial t} = - \frac{\partial q_t}{\partial x}
\]

Exner’s seminal contribution: if more sediment enters a reach than leaves, the bed elevation in the reach increases.

Relation between sediment transport rate and flow hydraulics:

\[
q_t = q_t(U) = \alpha U^n, \quad n > 0
\]

The phenomenon of sediment transport was poorly known in Exner’s time. Exner guessed that a higher velocity caused a higher sediment transport rate.
REDUCTION

\[ \text{UH} = q_w \quad \text{and} \quad H = \xi - \eta \quad \Rightarrow \quad U = \frac{q_w}{\xi - \eta} \]

\[ \frac{\partial U^2 H}{\partial x} = -\frac{1}{2} g H \frac{\partial H}{\partial x} - \frac{1}{2} g H \frac{\partial \eta}{\partial x} \quad \text{and} \quad H = \xi - \eta \quad \text{and} \quad U = \frac{q_w}{\xi - \eta} \]

\[ \Rightarrow \]

\[ \frac{\partial H}{\partial x} = -\frac{1}{(1 - \text{Fr}^2)} \frac{\partial \eta}{\partial x} \quad \text{and} \quad \frac{\partial \xi}{\partial x} = -\frac{\text{Fr}^2}{(1 - \text{Fr}^2)} \frac{\partial \eta}{\partial x} \]

where

\[ \text{Fr} = \frac{U}{\sqrt{g H}} = \text{Froude number} \]

Range for dunes: low Froude number: \( \text{Fr}^2 << 1 \)

\[ \frac{\partial \xi}{\partial x} = -\frac{\text{Fr}^2}{(1 - \text{Fr}^2)} \frac{\partial \eta}{\partial x} \approx 0 \quad \therefore \quad \text{Constant water surface elevation} \]
MORE REDUCTION

\[ U = \frac{q_w}{\xi - \eta} \quad \text{and} \quad q_t = \alpha U^n \quad \text{and} \quad \frac{\partial H}{\partial x} = -\frac{1}{1 - Fr^2} \frac{\partial \eta}{\partial x} \]

and \( Fr^2 \ll 1 \) and \( \xi = \text{constant} \)

substituted into

\[ (1 - \lambda_p) \frac{\partial \eta}{\partial t} = -\frac{\partial q_t}{\partial x} \]

yields

\[ \frac{\partial \eta}{\partial t} + c(\eta) \frac{\partial \eta}{\partial x} = 0 \]

Since \( \xi = \text{constant} \), \( q_w = \text{constant} \) and \( n > 0 \),

\[ c > 0 \text{ is an increasing function of } \eta! \]
THE RESULT

Dunes migrate downstream, and migration speed increases with bed elevation

\[ \frac{\partial \eta}{\partial t} + c(\eta) \frac{\partial \eta}{\partial x} = 0 \]

And thus the asymmetry!

Exner’s original sketch
The field of **sediment morphodynamics** consists of the class of problems for which the flow over a bed interacts strongly with the shape of the bed, both of which evolve in time.

\[
\begin{align*}
\frac{\partial H}{\partial t} & + \frac{\partial UH}{\partial x} = 0 \\
\frac{\partial UH}{\partial t} + \frac{\partial U^2H}{\partial x} & = - \frac{1}{2} gH \frac{\partial H}{\partial x} - gH \frac{\partial H}{\partial x} \\
\end{align*}
\]

Quasi-steady assumption:
The flow naturally talks fast, but can also talk slow. The bed naturally talks slow. The only part of the flow’s talk that the bed hears is the slow part. (Quasi-steady assumption: \(q_t/q_w << 1\))

\[
q_t = q_t(U)
\]

\[
(1 - \lambda_p) \frac{\partial \eta}{\partial t} = - \frac{\partial q_t}{\partial x}
\]
Could you slow down a bit, I’m having a bit of trouble trying to get the gist of it all.
SCALES

Paraná delta, Argentina
10 ~ 100 km

Buenos Aires

Delta advances
~ 300 m/year

Current ripples:
~ 20 cm wavelength

https://zulu.ssc.nasa.gov/mrsid/

http://www.ux1.eiu.edu/~cfips/1300/ripples.jpg
LONGITUDINAL STREAKS

Image courtesy T. Tsujimoto
LONGITUDINAL STREAKS:
LINEAR STABILITY ANALYSIS

\[ V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = 1 + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}, \]
\[ V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = -\frac{\partial P}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}, \]
\[ V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = -\frac{\partial P}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}, \]
\[ \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0, \]
\[ \tau_{ij} = -\frac{2}{3}k\delta_{ij} + 2\nu_t D_{ij} + C_D l^2 (D_{im} D_{mj} - \frac{1}{3} D_{mn} D_{mn} \delta_{ij}) + C_E l^2 (\tilde{D}_{ij} - \frac{1}{3} \tilde{D}_{mn} \delta_{ij}). \]

Here \( D_{ij} \) is the mean rate of strain tensor, \( \nu_t = \frac{1}{2} k^{1/2} l \) is the eddy viscosity and

\[ \tilde{D}_{ij} = \frac{D}{Dt} - \frac{\partial U_i}{\partial x_k} D_{kj} - \frac{\partial U_j}{\partial x_k} D_{ki} \]
DUNES, ANTIDUNES

Image courtesy D. Mohrig
DEFINITION OF DUNES AND ANTIDUNES

Dunes are 1D (or quasi-1D) bedforms for which the water surface fluctuations are approximately *out of phase* with the bed fluctuations. That is, the water surface is high where the bed is low and vice versa. As is shown below dunes migrate downstream.

Antidunes are 1D (or quasi-1D) bedforms for which the water surface fluctuations are approximately *in phase* with the bed fluctuations. That is, the water surface is high where the bed is high and vice versa. As shown below, most antidunes migrate upstream, but there is a regime within which they can migrate downstream.
REGIME DIAGRAM: POTENTIAL FLOW OVER A WAVY BED

\[ \eta_0 = \text{amplitude of bed perturbation} \]
\[ H_o = \text{unperturbed depth} \]

Linearized potential flow analysis is sufficient to explain existence regimes, but not formation (gives neutral stability)

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - g \frac{\partial \eta}{\partial x} \]
\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial v} - g \]

\[ \eta = \eta_0 \sin(kx), \quad k = \frac{2\pi H_o}{\lambda} \]
PHASE DIAGRAM FOR DUNES AND ANTIDUNES BASED ON LINEAR POTENTIAL THEORY OVER A WAVY BED

\[ Fr_0 = \frac{1}{\sqrt{k \tanh(k)}} \]

\[ Fr_0 = \sqrt{\frac{\tanh(k)}{k}} \]

- **Subcritical response** (dunes possible)
- **Supercritical response** (antidunes possible)
- **Long wave limit**
- **Downstream-migrating**
- **Upstream-migrating**
FLOW IN THE DUNE REGIME

\( Fr_o < [\tanh(k)/k]^{1/2} \)
\( k = 2\pi H/\lambda \)
\( H = \text{depth}, \lambda = \text{wavelength} \)

Water surface is out of phase with the bed.
Depth variation is out of phase with the bed
Flow accelerates from trough to crest.
Sediment transport increases from trough to crest.
Bedform migrates downstream.
Bedform becomes asymmetric.

\[
k \to 0 \quad \text{(shallow water)}
\]
\[
\frac{\partial H}{\partial x} = -\frac{1}{(1 - Fr^2)} \frac{\partial \eta}{\partial x}
\]
\[
\frac{\partial \xi}{\partial x} = -\frac{Fr^2}{(1 - Fr^2)} \frac{\partial \eta}{\partial x}
\]
FLOW IN THE UPSTREAM-MIGRATING ANTIDUNE REGIME

\([\tanh(k)/k]^{1/2} < Fr_o < [k \tanh(k)]^{-1/2}\]

Water surface is in phase with the bed.
Depth variation is in phase with the bed.
Flow decelerates from trough to crest.
Sediment transport decreases from trough to crest.
Bedform migrates upstream (or hardly at all).
Bedform stays symmetric.

\[
\frac{\partial H}{\partial x} = -\frac{1}{(1-Fr^2)} \frac{\partial \eta}{\partial x}
\]

\[
\frac{\partial \xi}{\partial x} = -\frac{Fr^2}{(1-Fr^2)} \frac{\partial \eta}{\partial x}
\]

\(k \rightarrow 0\) (shallow water)
FLOW IN THE DOWNSTREAM-MIGRATING ANTIDUNE REGIME

\[ [k \tanh(k)]^{-1/2} < F_{r_0} \]

Water surface is in phase with the bed.
Depth variation is out of phase with the bed.
Flow accelerates from trough to crest.
Sediment transport increases from trough to crest.
Bedform migrates downstream.
Bedform becomes asymmetric.
*These are antidunes that look like dunes: not too common, but they are observed.*

No shallow-water limit as \( k \to 0 \).
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]
\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} - g \frac{\partial \eta}{\partial x} + \frac{\partial}{\partial x} \left( v_t \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial x} \left( v_t \frac{\partial u}{\partial y} \right) - g \frac{\partial \eta}{\partial x} \]
\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial v} - g + \frac{\partial}{\partial x} \left( v_t \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial x} \left( v_t \frac{\partial v}{\partial y} \right) - g \frac{\partial \eta}{\partial x} \]
\[ \eta = \eta_0 e^{\alpha t} \sin(kx) \quad , \quad k = \frac{2\pi H_o}{\lambda} \]

Closure for \( v_t \): a constant value that gives a result close to the logarithmic law

\[ (1 - \lambda_p) \frac{\partial \eta}{\partial t} = -\frac{\partial q_t}{\partial x} \]
\[ q_t = f(\tau_b, \frac{\partial \eta}{\partial x}) \]

In sediment transport law, \( \tau_b = \text{bed shear stress} \)
INSTABILITY MECHANISM FOR DUNES

Consider flow into a Venturi contraction. The favorable pressure gradient on the upstream side intensifies the bed shear stress. The adverse pressure gradient on the downstream side suppresses shear stress.

Wall shear stress $\tau_b$ intensified

Wall shear stress $\tau_b$ suppressed

Bed perturbation: black solid

Shear stress perturbation $\rightarrow$ sediment transport rate perturbation: red dashed

Sediment transport rate peaks a little before the bed perturbation peak
The bed shear stress perturbation, and thus the sediment transport rate perturbation, lead the bed elevation perturbation. There is thus net deposition at the apex (and net erosion at the trough), and so amplitude increases in time.

A nonlinear analysis including flow separation on the lee side of dunes is necessary to explain nonlinear equilibrium: numerical, e.g. $k$-$\varepsilon$. 

**NET DEPOSITION AT APEX: LINEAR MODEL**
SINGLE-ROW AND MULTIPLE-ROW ALTERNATE BARS

Occam’s razor minimal analysis:
2D shallow water equations +
2D sediment transport formulation

Controlling parameter: width-depth ratio B/H
No bars → single-row bars → multiple-row bars
NONLINEAR INTENSIFICATION OF SEDIMENT TRANSPORT RATE AT CONFLUENCES: SCOUR
DOWNSTREAM, THE FLOW EXPANDS AND DEPOSITS A MINI-FAN

Sunwapta River Canada

Image courtesy P. Ashmore
DOWNSTREAM, THE FLOW EXPANDS AND DEPOSITS A MINI-FAN

Sunwapta River Canada

Image courtesy P. Ashmore
AS THE FLOW GETS WIDER AND SHALLOWER, IT BECOMES UNSTABLE AND BIFURCATES INTO ONE OR MORE CHANNELS

Image courtesy P. Ashmore
AS THE FLOW GETS WIDER AND SHALLOWER, IT BECOMES UNSTABLE AND BIFURCATES INTO ONE OR MORE CHANNELS

Image courtesy P. Ashmore
MEANDERING

Mississippi River, USA

Image courtesy Y. Shimizu

From maps of H. Fisk

I forgot where
MEANDERING MECHANISM

Occam’s razor first analysis:

2D shallow-water equations corrected for effect of helical flow in bends (2.5D formulation)

Relation for channel migration:

\[ \dot{n}_o = E \Delta u \]

Locus of high streamwise flow velocity
ONLY BRAIDING IS POSSIBLE IN THE ABSENCE OF BANK STABILIZATION

Braided stream on the North Slope, Brooks Range, Alaska
SWEET LITTLE LIES

Yes, honey, I’m with you.
Inside bank to outside bank:

“Yes, I’m following you.”
Inside bank to outside bank:

“Yes, I’m following you.”
Yeah, while you were pretending to listen, *look at the mess we got ourselves into!*
BEND SKEWING: SUBCRITICAL BIFURCATION OF NONLINEAR STABILITY ANALYSIS

Bends grow until cutoff:
There is no nonlinear stable state

Old oxbow lake due to cutoff
SLUMPING ON OUTSIDE SLOWS DOWN EROSION SO TRAPPING OF SEDIMENT BY VEGETATION ON INSIDE CAN KEEP UP
I slump

Do we talk to each other?

I trap

Vermilion River, USA
NOW FOR A TOUR OF MORPHODYNAMIC PHENOMENA
WITHOUT DETAIL AS TO HOW THEY ARE SOLVED

Yes, they are tractable to various degrees
SCROLL BARS

I still can’t remember

Image courtesy Y. Shimizu

Strickland River, Papua New Guinea

Image courtesy W. Dietrich
ALLUVIAL FANS AND FAN-DELTAS

Selenga River at Lake Baikal, Russia

https://zulu.ssc.nasa.gov/mrsid/
THE OKAVANGO INLAND FAN, BOTSWANA, AFRICA

Graben: subsidence

https://zulu.ssc.nasa.gov/mrsid/
THE FAN-DELTA OF THE KUROBE RIVER, JAPAN

https://zulu.ssc.nasa.gov/mrsid/
THE FAN-DELTA OF THE IOCC IRON MINE, LABRADOR, CANADA
THE FAN IN THE DELTA

http://wakamononoh.logoz.org/images/toimg3.jpg
FANS AND FAN-DELTAS AT VARIOUS SCALES

Laboratory fan-delta, ~ 3 m.
Image taken at St. Anthony Falls Laboratory, University of Minnesota USA.
Fan created by runoff from cultivated field; ~ 6 m. Image taken by author near Pigeon Point, California.
Fan in Idaho, USA created by runoff from burned hillside, ~ 50 m.
FANS AND FAN-DELTAS AT VARIOUS SCALES contd.

Copper Creek Fan, Death Valley, USA; ~ 10 km. Image courtesy Roger Hooke.
FANS AND FAN-DELTAS AT VARIOUS SCALES contd.

Kosi River Fan, India; ~ 125 km.

https://zulu.ssc.nasa.gov/mrsid/
RIVER MIGRATION AND AVULSION MAKES FANS

Yellow River Fan-delta, China

https://zulu.ssc.nasa.gov/mrsid/
CONCAVE BANK
BENCHES

Fly River, Papua New Guinea

Image courtesy OTML
SELF-CHANNELIZATION: NATURAL LEVEES

Image courtesy National Geographic

Gilgal Abey River, Ethiopia

Mississippi River, USA

https://zulu.ssc.nasa.gov/mrsid/
SEA LEVEL ROSE SOME 120 M SINCE THE END OF THE LAST GLACIATION

How does a river mouth respond to sea level rise?
• Does a delta continue to prograde into the ocean?
• Or does the sea drown the delta and invade the river valley (transgression)?
DELTAS AND SEA LEVEL RISE

Experiment on effect of base level rise on delta

Image courtesy T. Muto
HOW DID THE DELTAS OF MAJOR RIVERS RESPOND?

Delta of the Paraná River, Argentina

https://zulu.ssc.nasa.gov/mrsid/
Somewhere in Bolivia

https://zulu.ssc.nasa.gov/mrsid/
SUBMARINE MORPHODYNAMICS DUE TO TURBIDITY CURRENTS

California Margin

Image courtesy MBARI
CANYON EXCAVATION

Monterey submarine canyon

Image courtesy MBARI
MEANDERING OF SUBMARINE CHANNELS

Mississippi Submarine Fan (Weimer, 1991).

Indus Submarine Fan (Kenyon et al., 1995)

Amazon Submarine Fan (Pirmez, 1995)
CONGO DEEP-SEA FAN

EM12 Bathymetry of the Pleistocene to Present Zaire Deep-Sea Fan.
FANS AND CANYONS: STEPPED PROFILES

Stepped profile, Niger Margin
From Prather et al. (2003)
SELF-CONFINEMENT AND LEVEE CONSTRUCTION

Turbidity currents are adept at confining themselves between levees.

Channel on Amazon Submarine Fan
Damuth and Flood (1985)

Toyama Submarine Channel
Kubo and Nakajima (2002)
SELF-CONTAINMENT

Submarine meandering channels contain themselves between levees over 100’s ~ 1000’s of km and scores ~ 100’s of bends.


Zaire Fan: Savoye, Cochonat et al. (2000)
CYCLIC STEPS:
A UNIVERSAL BEDFORM OF FROUDE-SUPERCritical FLOW
IN RIVERS AND TURBIDITY CURRENTS FLOWING
OVER ERODIBLE BEDS

Dry Meadow Creek, USA
Small stream near Calais, France
Deep sea offshore of California, USA

Images courtesy M. Neumann, H. Capart and L. Pratson
Trains of cyclic steps in a coastal outflow channel on a beach in Calais, France. Image courtesy H. Capart.

The steps move upstream.
THE IDEA

Steady, uniform (normal) Froude-supercritical flow \((Fr_n > 1)\) over a freely-erodible bed of sand might be unstable,

and within an appropriate range might not devolve to ephemeral, short-wave \((L/h \sim 1)\) antidunes, but instead would devolve to

orderly, sustained trains of long-wave \((L/h << 1)\) cyclic steps, with regions of subcritical and supercritical flow bounded by hydraulic jumps.
Sufficiently supercritical flow over a plane bed is subject to a *long-wave* instability that devolves into upstream-migrating supercritical and subcritical regions bounded by hydraulic jumps.
LET’S LOOK AT THIS IMAGE AGAIN

Roll waves

Cyclic steps!

Tailings Fan, Lake Wabush, Canada
THE SAME CYCLIC STEP INSTABILITY IS FOUND IN INCISING BEDROCK STREAMS

Images courtesy Michael Neumann, Gough Island Weather Station and Ellen Wohl
CYCLIC STEPS IN BEDROCK:
This one is too beautiful not to show

Ojiro River, Japan
WHAT ARE THESE WASHBOARD-LIKE FEATURES IN THE DEEP SEA?

Seabed “sediment waves” off the California margin

Image courtesy MBARI
SUBAQUEOUS DEPOSITIONAL AND EROSIONAL CYCLIC STEPS
EARTH → TITAN

Water → liquid methane
Granitic rock → ice as a “rock”
The evidence suggests that at least near where Huygens touched down, there is a *plethora of alluvium in the gravel and sand sizes*. The gravel presumably consists of *water ice* and appears to be *fluvially rounded*.\(^{95}\)
AN EXCITING FUTURE WAITS