Optimal Vortex Formation as a Unifying Principle in Biological Propulsion

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BIOENGINEERING
Vortex Rings

A Secondary Eruption of Mt. St. Helens, June 1980 [Photo by Robert P. VanNatta]

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Zebrafish \((Danio rerio)\)
Embryo

Koster, Forouhar and Gharib (2002)
Previous Work on Vortex Rings

• Early work: Helmholtz (1858), Kelvin (1869)
• Existence: Hill (1894), Fraenkel (1972), Norbury (1973), and others
• Evolution and Turbulent rings: Maxworthy (1972, 1974, 1977), Glezer (1987), and others

Fully-Pulsed Jets:

• Bremhorst et al. (e.g., 1979, 1990, and 2000)
• Weihs (1977)
Classical View

Vortex ring “…formation is a problem of vortex sheet dynamics, the steady state is a problem of existence, their duration is a problem of stability, and if there are several we have a problem of vortex interactions.”

-- P.G. Saffman (1981), emphasis added
“New” Motivations

**Understanding complicated biological flows:**
- Aquatic Propulsion
- Cardiac Flows

**Practical Applications:**
- Hydropropulsion /Aeropropulsion
- Micro jet thrusters
- Multi-scale Stirring and Mixing
Canonical Vortex Ring Generator

Vortex rings can be easily generated using a piston-cylinder mechanism to produce a starting jet.

Parameters:

a) Time history of piston velocity
b) L/D
c) Reynolds Number
d) Orifice/nozzle Geometry

Can be viewed as the roll up of a half-"infinite" cylindrical vortex sheet.
Vortex Ring Formation

Vortex Ring Formation
# Vortex Ring Experiments

<table>
<thead>
<tr>
<th>Author</th>
<th>Year</th>
<th>L/D</th>
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<tbody>
<tr>
<td>Glezer</td>
<td>1981</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Didden</td>
<td>1979</td>
<td>&lt;2</td>
</tr>
<tr>
<td>Kwon and Bernal</td>
<td>1989</td>
<td>&lt;4</td>
</tr>
<tr>
<td>Auerbach</td>
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<td>Schatzle</td>
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<td>Weigand and Gharib</td>
<td>1994</td>
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<tr>
<td>Maxworthy</td>
<td>1977</td>
<td>&lt;3</td>
</tr>
<tr>
<td>Sallet</td>
<td>1974</td>
<td>&lt;1</td>
</tr>
</tbody>
</table>
Can we generate arbitrarily large vortex rings?

\[ \frac{L}{D} = \frac{\int_0^t U(\tau) \, d\tau}{D} = 2 \]
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\[ L/D = \left[ \frac{\int_0^t U(\tau) \, d\tau}{D} \right] / D = 2 \]

\[ L/D = 3.8 \]
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\[ L/D = 3.8 \]

\[ L/D = 14.5 \]

\[ L/D \approx 4 \equiv \begin{cases} 
\text{Vortex ring formation time} \\
\text{Vortex pinch-off time}
\end{cases} \]

L/D = 2 vs. 4

L/D = 2

L/D > 4
The Formation Number ($F$)

Formation Time $\leftrightarrow \overline{U}_p t / D = L / D$

Models for Vortex Ring Pinch-off

- Gharib et al. (1998): invokes Kelvin-Benjamin Variational principle

Vortex ring pinch off “...occurs when the source energy falls below that of a steadily translating vortex ring”

\[ U_j < U_v \]

Vortex ring growth is limited by energy effects

For vortex ring growth: vortex generator energy > vortex ring energy

(W.T. Kelvin, 1875; T.B. Benjamin 1976)

\[ \alpha = \frac{E/\rho}{\sqrt{(I/\rho)\Gamma^3}} \]

dimensionless energy vs. \( L/D \)

Vortex ring growth is limited by energy effects

For vortex ring growth: \( \text{vortex generator energy} > \text{vortex ring energy} \)

(W.T. Kelvin, 1875; T.B. Benjamin 1976)

\[
\begin{align*}
\text{dimensionless energy} & \quad \uparrow \\
L/D & \quad \downarrow \\
\hline
\text{steady vortex ring energy} & \\
\end{align*}
\]
Vortex ring growth is limited by energy effects

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\[ L/D \]

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Models for Vortex Ring Pinch-off

\[
\alpha \equiv \frac{E/\rho}{\sqrt{(I/\rho)\Gamma^3}}
\]

\[\alpha_{\text{piston}} \quad \alpha \quad \alpha_{\text{limiting}} \quad \alpha_{\text{Impulse}} \quad \alpha_{\text{Fast ramp}} \quad \alpha_{\text{Slow ramp}} \]

Models for Vortex Ring Pinch-off


\[ U_{vortex} = 0.5 U_{piston} \]


\[ \Omega_{jet} = \Omega_{vortex} \]
Models for Vortex Ring Pinch-off

• Mohseni (1998): combines Norbury vortex model and slug model approximation for ring translational velocity

\[ U_{vortex} = 0.5 U_{piston} \]

• Linden and Turner (2001): combines Norbury vortex model and volume conservation approximation

\[ \Omega_{jet} = \Omega_{vortex} \]

However, entrained fluid by the leading vortex can reach over 50 percent of its total volume (\( \Omega_{vortex} \))


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Physical Implications of Pinch-Off

Pinch-Off $\Rightarrow$ Maximum vortex ring strength (energy)

$\Rightarrow$ Maximum fluid entrainment per vortex ring and maximum vortex ring velocity

$\Rightarrow$ Maximum thrust per pulse
Total Impulse of Starting Jets

\[ I_U = I_U(t_p) = \int_0^{t_p} \int_A u_j^2(r, \tau) dA d\tau \]

Average Force of Starting Jets


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Added and Entrained Mass

\[ I_U + I_p = \left( m_{\text{ejected}} + m_{\text{entrained}} + m_{\text{added}} \right) W \]

Experiments \( \Rightarrow \) \( m_{\text{ejected}} W < I_U \)

Limiting physical processes dictate “optimal” parameters for vortex ring formation

Experiments demonstrate correlation between vortex ring pinch-off and maximum mass and momentum transfer

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Do biological systems exploit vortex ring formation for optimal fluid transport?
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J.O. Dabiri et al. (2004)
Previous attempts to verify optimal vortex formation in biological systems have been inconclusive.
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Measurements in the literature…

Squid: \[ \frac{L}{D} = 4-7, 10-40, 34, 87 \]
Salps: \[ \frac{L}{D} = 3-4, 6.7, 10-20 \]
Jellyfish: \[ \frac{L}{D} = 0.2-4.4 \]

Are we missing something?
Biological Factors Affecting Pinch-off

- Co-flow
  
  Swimming and flying animals generate vortices in a free-stream flow
Biological Factors Affecting Pinch-off

- **Co-flow**
  Swimming and flying animals generate vortices in a free-stream flow

- **Temporal exit diameter variation**
  e.g. squid (Bartol *et al.*, 2001)
  Parallels time-dependent flap kinematics in other locomotion modes
A proper analysis must include fluid-structure interactions

External interactions: Flow past the vortex generator

Flow past the propulsor can affect vortex ring formation
A proper analysis must include fluid-structure interactions

External interactions: Flow past the vortex generator

Effects of Co-flow

\[ F = \frac{(U_{\text{external}} + U_{\text{jet}})T}{D} \]
\[ R_v = \frac{U_{\text{external}}}{U_{\text{jet}}} \]

Effects of Co-flow

- $U_p = 11.4$ cm/s
- $U_p = 5.5$ cm/s

Bartol et al., 2001

Dabiri et al., 2004
Effects of Co-flow

Animals can exploit external flow in other flow-related functions (i.e. feeding and maneuvering)

See Gallery of Fluid Motion Video #14 (Dabiri et al.)

feeding via vortex ring fluid entrainment

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A proper analysis must include fluid-structure interactions

Internal interactions: Dynamical effects of a variable exit $D(t)$
A proper analysis must include fluid-structure interactions.

Internal interactions: Dynamical effects of a variable exit $D(t)$

Possible effects

1) As a source of additional vorticity
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2) As a manipulator of existing vorticity
A proper analysis must include fluid-structure interactions

Internal interactions: Dynamical effects of a variable exit $D(t)$

Possible effects

1) As a source of additional vorticity
2) As a manipulator of existing vorticity

These effects are obscured when the time-dependent $D(t)$ is replaced by $\overline{D}$
A new technique replicates fluid-structure interactions in variable-diameter jet flows
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A new index for vortex formation properly accounts for time-dependent boundary conditions

“DYNAMIC FORMATION TIME”

Increase the vortex formation time incrementally:

$$\Delta \frac{L}{D}^* \equiv \left( \frac{U(\tau)}{D(\tau)} \right) \Delta \tau$$
A new index for vortex formation properly accounts for time-dependent boundary conditions

“DYNAMIC FORMATION TIME”

Increase the vortex formation time incrementally:

$$\Delta (L/D)^* \equiv (U(\tau)/D(\tau))\Delta \tau$$

Integrating from flow initiation at $$\tau = 0$$ to flow termination at $$\tau = t$$:

$$\frac{L}{D}^* = \int_{0}^{t} (U(\tau)/D(\tau))d\tau$$

Nondimensional Analysis

- **Normalized Circulation**

\[ \Gamma^* = \frac{\Gamma(U_e / D_e)}{U_e^2} \]

- **Normalized Time**

\[ T^* = \int_0^t \frac{U_e(\tau)}{D_e(\tau)} d\tau = \left( \frac{U_e}{D_e} \right) t \]

Vortex formation time is unaffected by temporal increases in jet diameter

- \((L/D)^* = 4.0 \pm 0.5\) for all \(dD(t)/dt > 0\) tested
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These results suggest the first set of engineering strategies for optimal fluid transport

During fluid ejection at $(L/D)^* < 4$:

$$I_{(L/D)^*<4} \sim D^2$$

(J.O. Dabiri & M. Gharib, *J Fluid Mech*)
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If total vortex formation time \((L/D)^* \rightarrow 4\), fluid is transported with maximum efficiency

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Are these strategies observed in biological systems?

Animal swimming revisited

- Squid rely on jet flow for high-speed swimming and escaping predation
Are these strategies observed in biological systems?

Animal swimming revisited

- Squid rely on jet flow for high-speed swimming and escaping predation through impulse maximization.
Are these strategies observed in biological systems?

Animal swimming revisited

- Squid rely on jet flow for high-speed swimming and escaping predation
  - **Impulse maximization**
Are these strategies observed in biological systems?

Animal swimming revisited

- Squid rely on jet flow for high-speed swimming and escaping predation
  - Impulse maximization

\[ \frac{D(t)}{D} \frac{dD(t)}{dt} > 0 \]

\[ \frac{D(t)}{D} \frac{dD(t)}{dt} < 0 \]

\[ (L/D)^* = 4 ? \]

- Transport complete?
  - Yes
  - Efficiency maximized
  - No

- Transport complete?
  - Yes
  - Impulse maximized
  - No

---


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Are these strategies observed in biological systems?

- The optimal ejection strategy depends on cardiac health efficiency and/or impulse maximization

\[
\frac{dD(t)}{dt} < 0 \quad \text{Transport fluid with } \frac{dD(t)}{dt} > 0
\]

\[(L/D)^* = 4 ? \]

- Transport complete?
  - yes
    - Efficiency maximized
  - no
    - Transport fluid with \( \frac{dD(t)}{dt} = 0 \)

- Efficiency maximized?
  - yes
    - Impulse maximized
  - no
    - Transport complete?
Are these strategies observed in biological systems?

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![Diagram showing the transport of fluid with $\frac{dD}{dt} > 0$ and $\frac{dD}{dt} < 0$.](image)

Are these strategies observed in biological systems?

- The optimal ejection strategy depends on cardiac health efficiency and/or impulse maximization.
Are these strategies observed in biological systems?

- The optimal ejection strategy depends on cardiac health efficiency and/or impulse maximization

\[
\frac{dD(t)}{dt} > 0 \quad \text{Transport fluid with } dD(t)/dt > 0 \\
\frac{dD(t)}{dt} < 0 \quad \text{Transport fluid with } dD(t)/dt < 0
\]

\[(L/D)^* = 4 ? \quad \text{yes} \quad \text{no} \]

- Transport complete?
  - yes
  - no

- Efficiency maximized
  - no
  - yes

- Impulse maximized
  - no
  - yes

Are these strategies observed in biological systems?

• The optimal ejection strategy depends on cardiac health efficiency and/or impulse maximization

\[ \frac{D}{D^*} = 4 \]

\[ \frac{L}{D^*} = 63 \]

Conclusions

Do biological systems exploit vortex ring formation for optimal fluid transport?
Conclusions

Do biological systems exploit

Yes, both mobile and
The vortex ring motif studied here is not limited to jet-based fluid transport…

**Flapping**

- birds (Rayner, 1988)
- fish (Drucker, 2000)

**Undulating**

- eels (Tytell, 2004)

**Paddling**

- frogs (Johansson, 2004)
...therefore, in order to better understand the physics and evolutionary incentives behind other vortex-based mechanisms, we need

To include realistic boundary and flow conditions such as compliance and co-flow

To investigate physics of individual events of vortex formation in the context of “Dynamic Formation Time” rather than Strouhal frequency
Acknowledgements

Special Thanks to

John O. Dabiri

And

Paul S. Krueger (Southern Methodist)

K. Mohseni (Colorado-Boulder), I. Bartol (Old Dominion), Michael Shusser (Technion), Moshe Rosenfeld (Tel Aviv), S. Colin (Roger Williams), J. Costello (Providence)

Supported by National Science Foundation and the Office of Naval Research