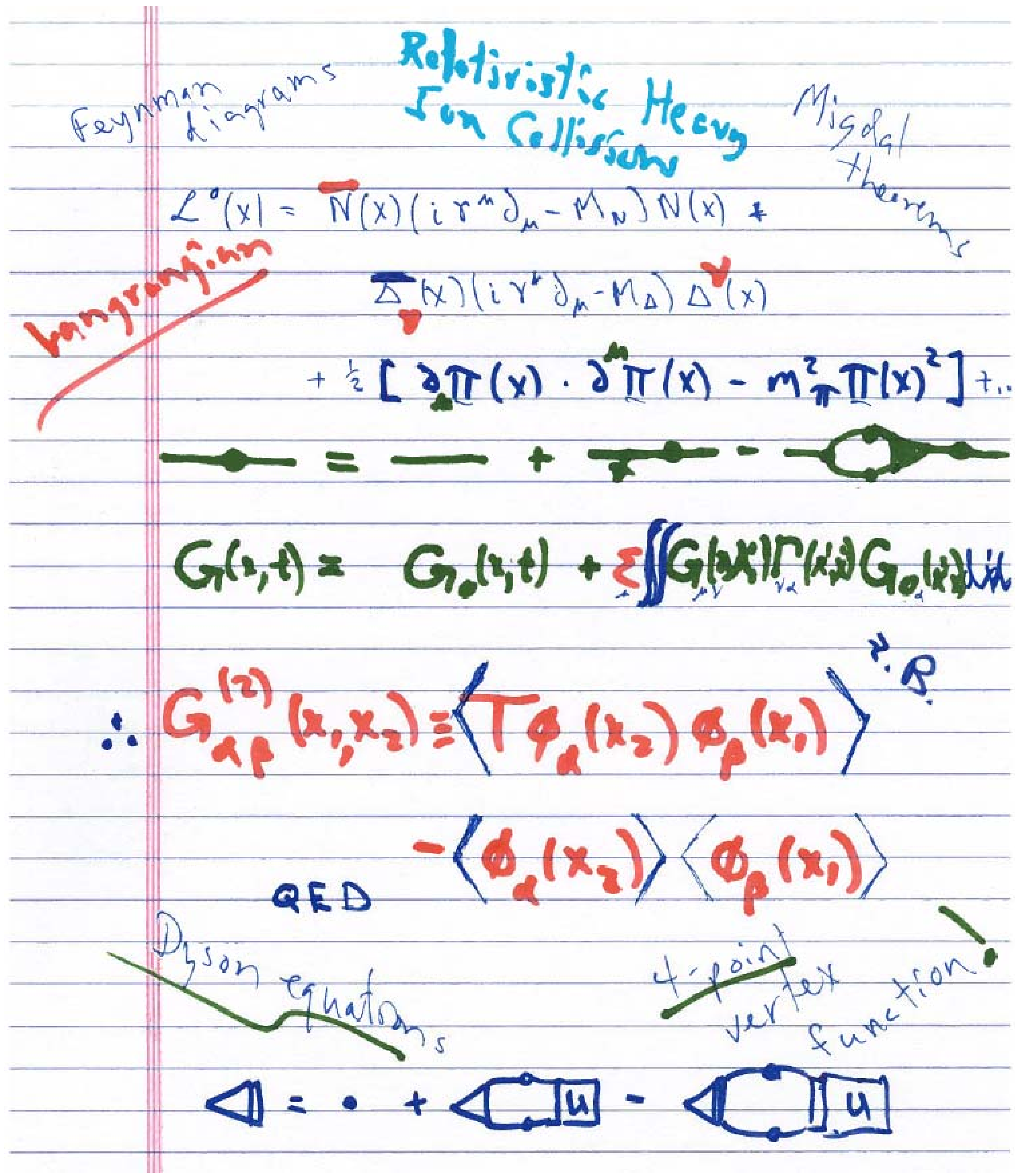
My first talk…

I thought I was supposed to derive the equations for the theory that we were developing for the professors in the room…

So it was 45 minutes of this →

I do remember using color to liven things up!

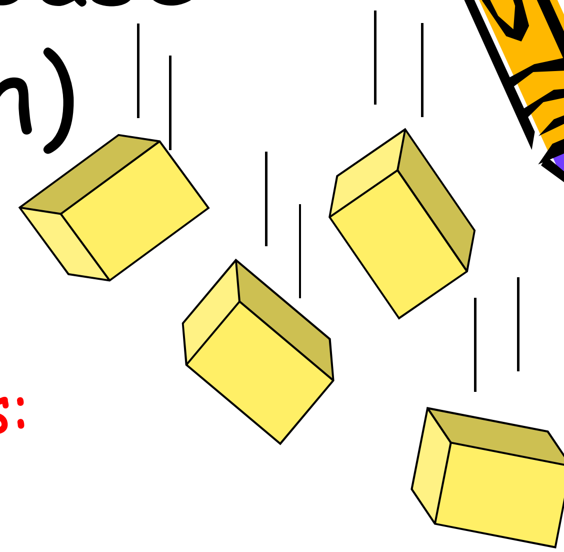


Unlikely Landings:

Dice, Coins, and the Mars Pathfinder

Gary White,
with Phoebe White, Susan White,
and many SPS members
Society of Physics Students and Sigma Pi Sigma
American Institute of Physics

Suppose you drop some boxes that have a square base (dimensions sxsxh)

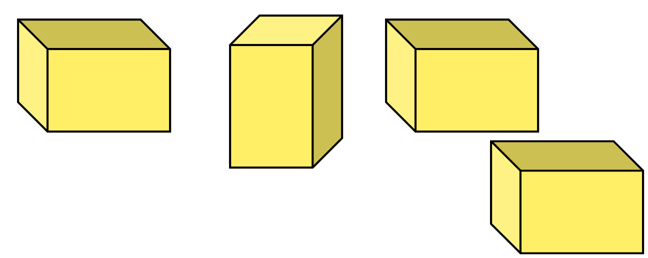


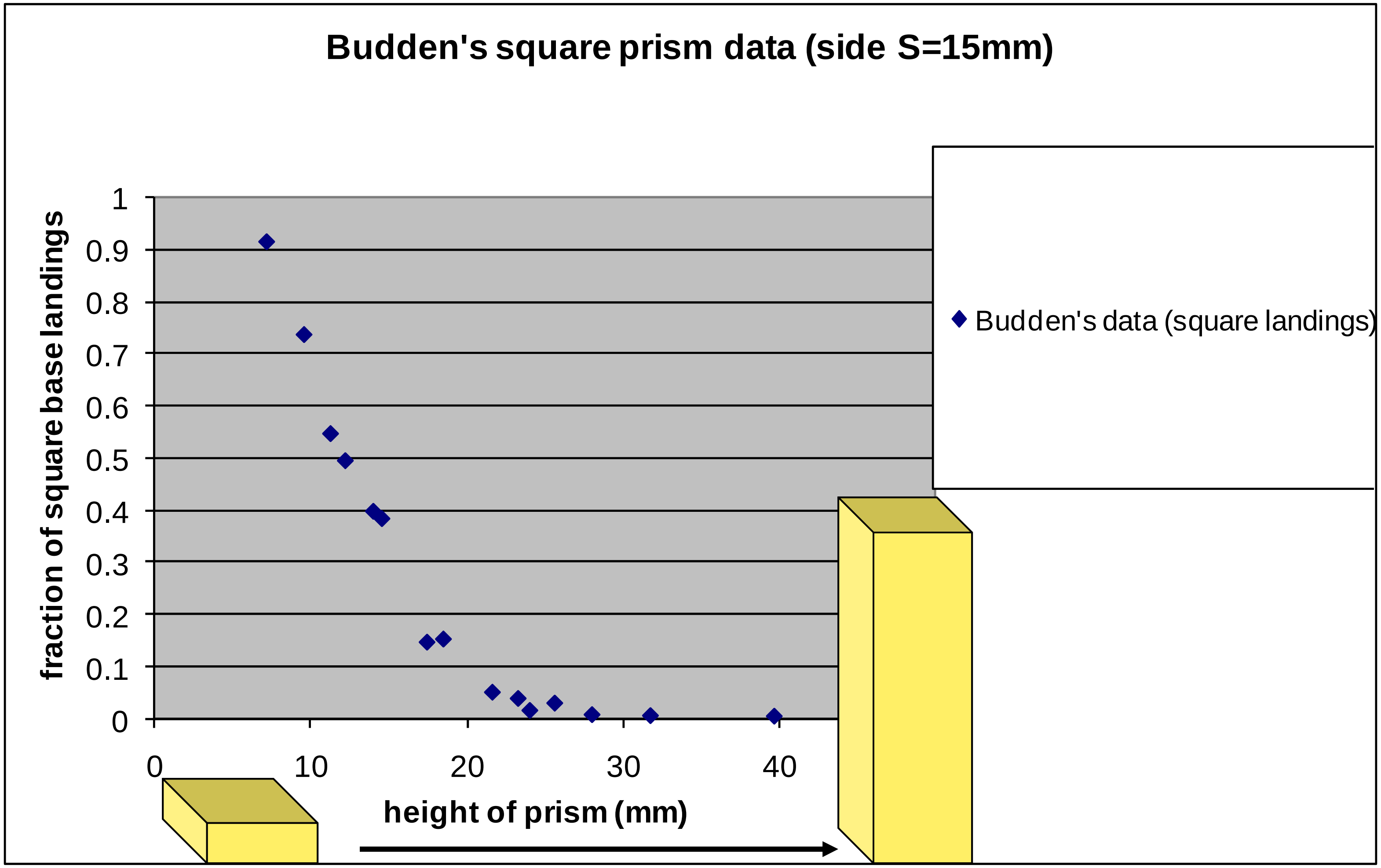
As you increase the height h, which do you think will be a better predictor for the number of square base landings:

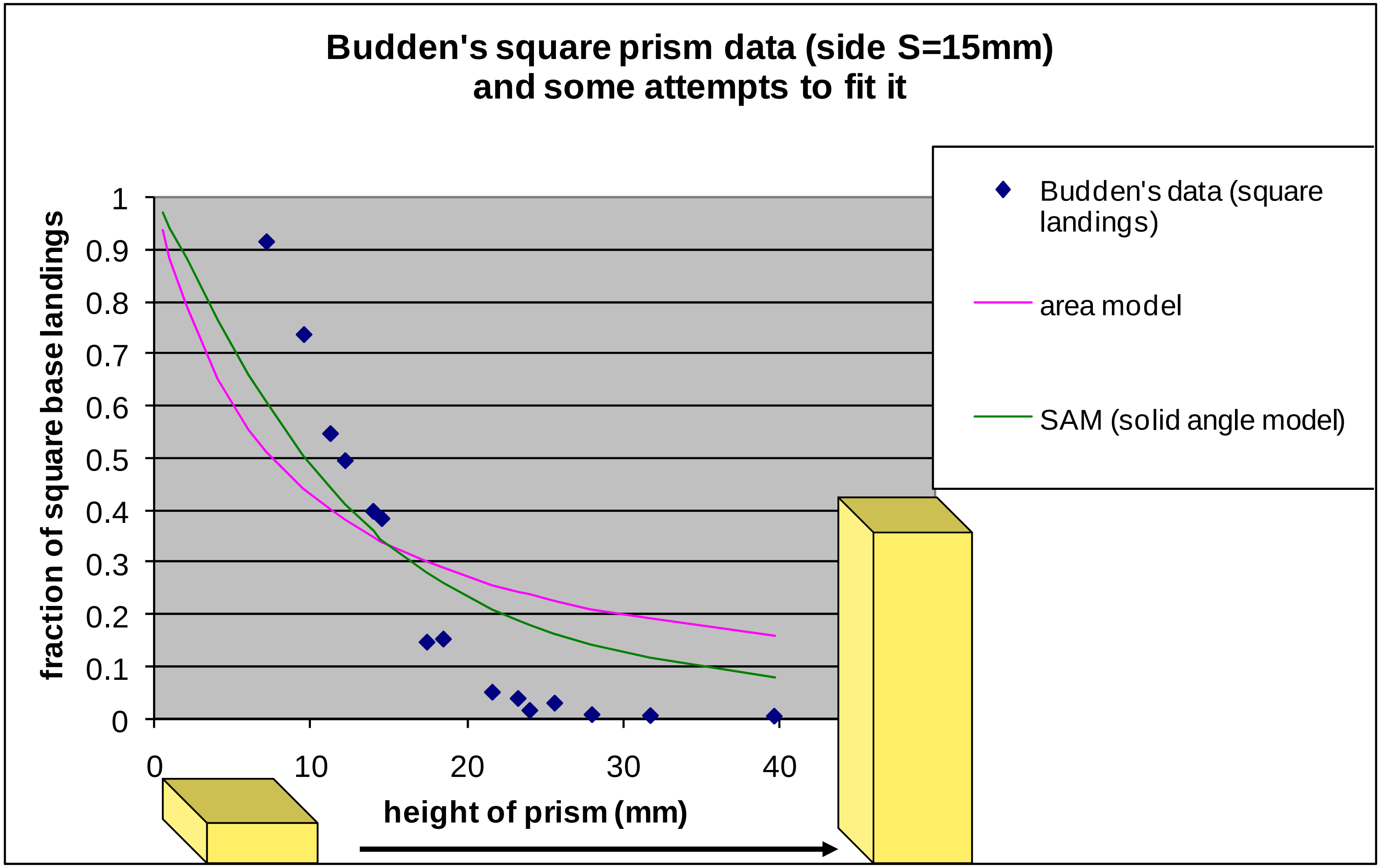
a) Area of the square as a fraction of total surface area?

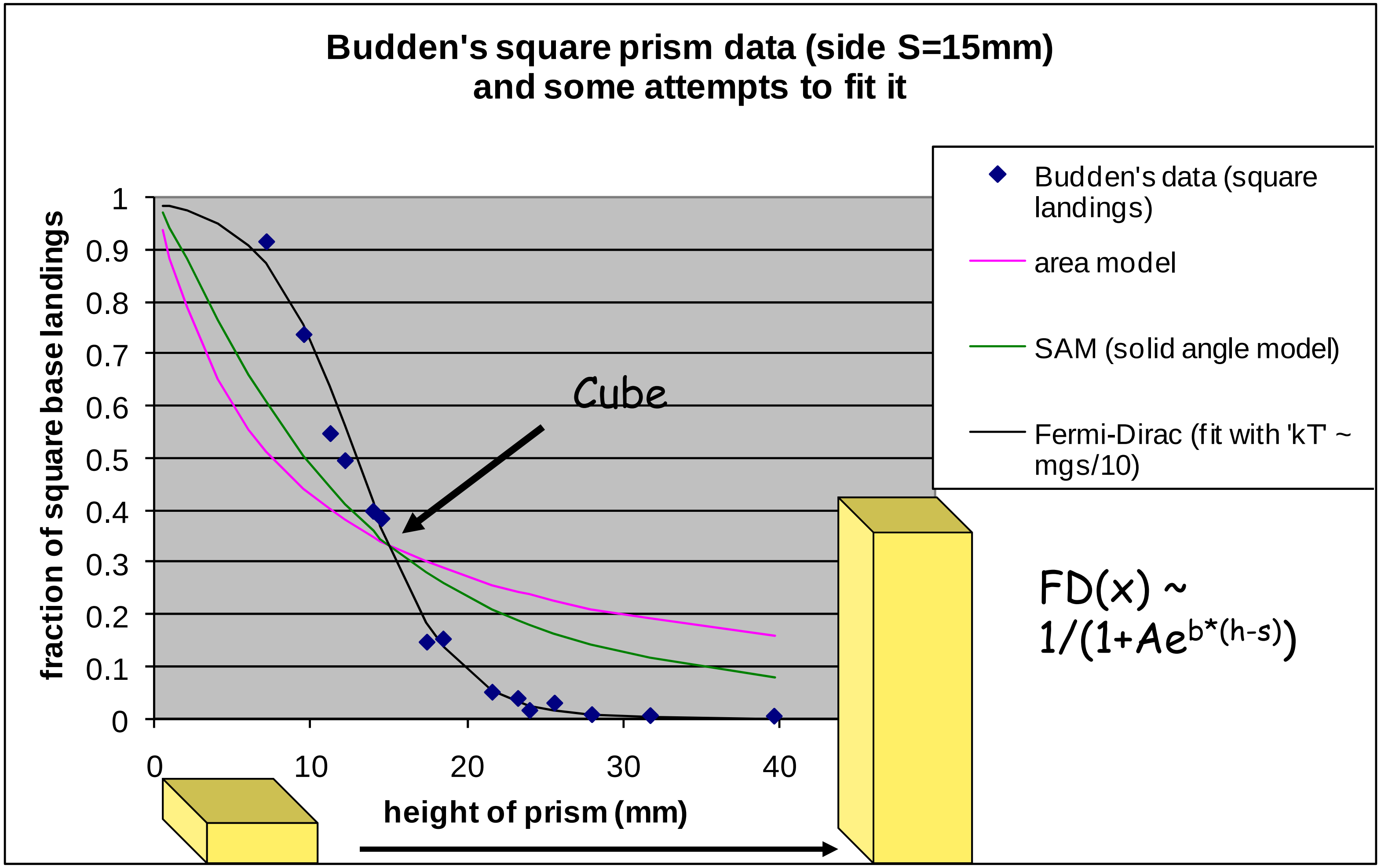
b) Solid angle subtended by the square from center of box?

c) The Fermi-Dirac distribution from quantum mechanics?




Budden's square prism data (side S=15mm)
fraction of square base landings
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
height of prism (mm)
Budden's data (square landings)


Budden's square prism data (side S=15mm)
and some attempts to fit it
fraction of square base landings
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
height of prism (mm)
Budden's data (square landings)
area model
SAM (solid angle model)

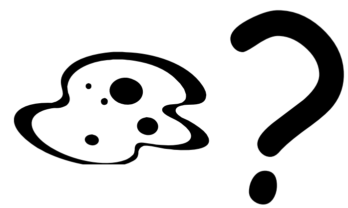


Surprised? I was…

Why study droppings?

Pedagogical reasons, general scientific curiosity, practical motivations…

- Math exercises for grammar school (F. Budden, D Singmaster in the *Mathematical Gazette*, Heilbronner, Berkshire, non-cube box dice, 1980-85)
- Gambling and loaded dice (E. Levin in AJP, brass slugs in plastic dice, 1983)
- Coins landing on edge (H. Bondi in EJP, D. Murray and S. Teare, in PhysRevE, 1993, and in Murray's thesis, 1991, building on work of Yue, Zhang, Keller, Vulovic, Prange, Feldberg and others)
- Complete list of fair dice (E. Pegg, UC at Colorado Springs master's thesis, 1997)
- And probability exercises for business students, undergraduate research, improved Mars landings (G. White, C. Gresham, D. Lutterman and many other students and SPS members, starting in 1992, still unpublished)

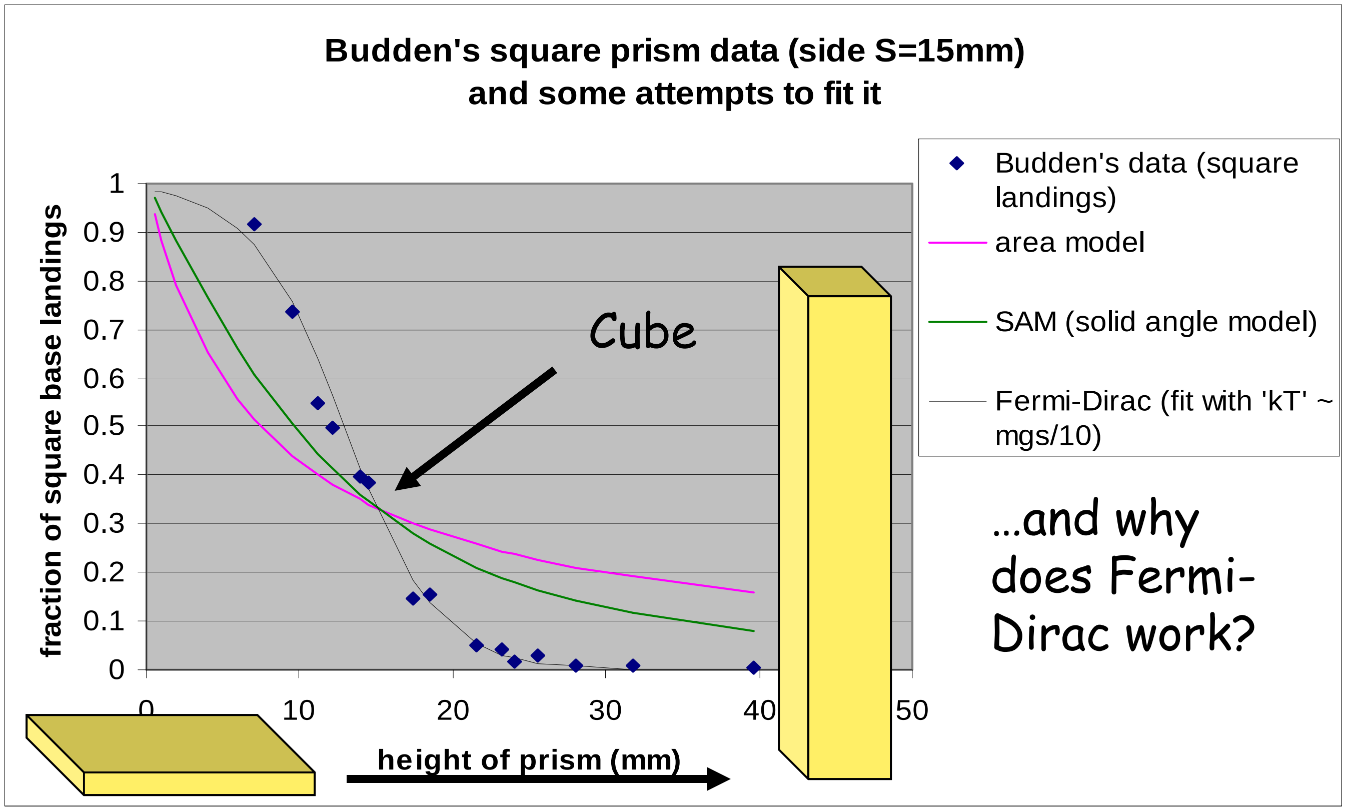


…improved Mars landings?

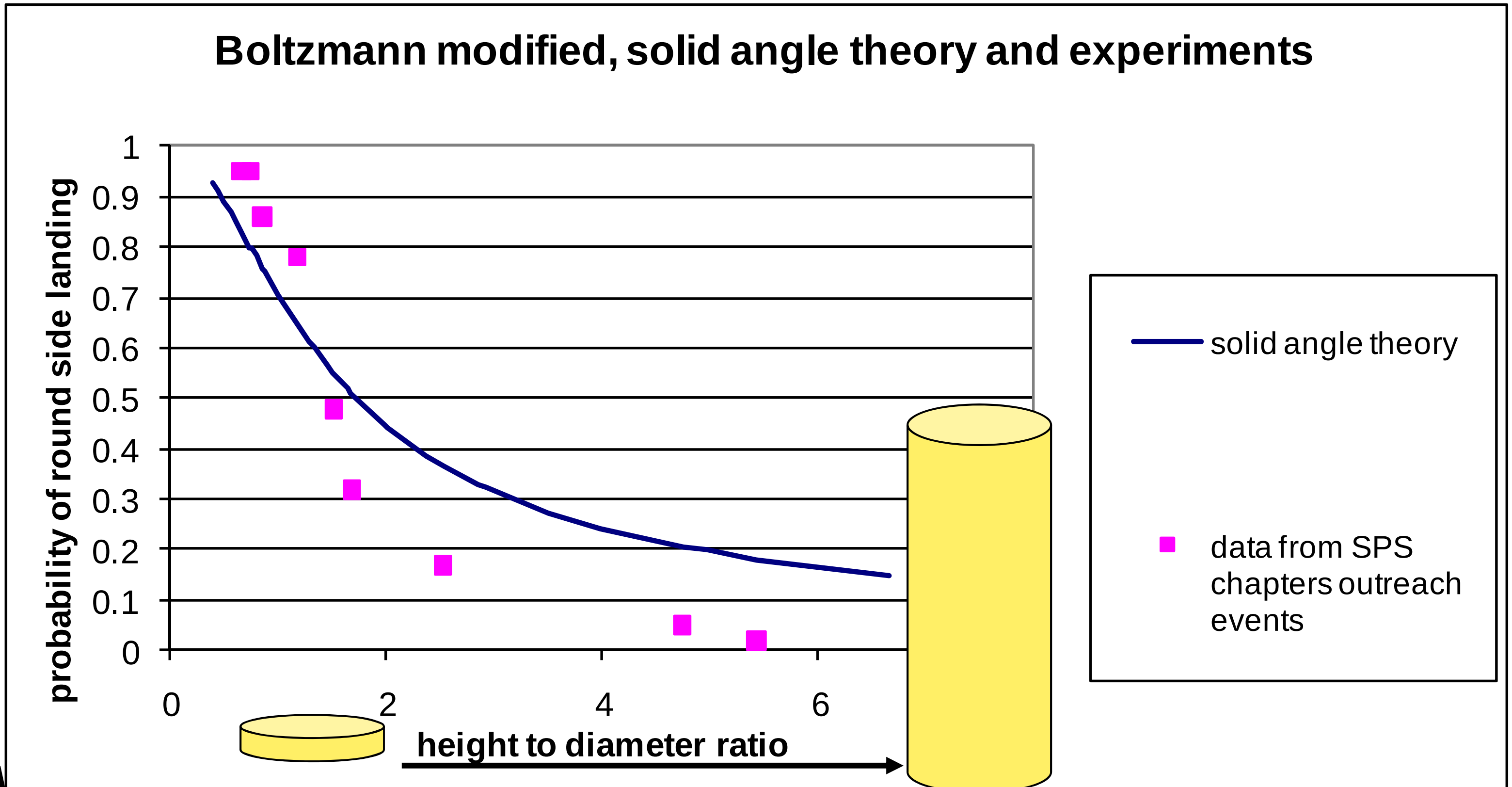
- In 1997, the Mars Pathfinder landed on its side.
- Turning motors then flipped it upright.
- If the science of landing is better understood, future landers could replace turning motors with more interesting scientific equipment.

As we saw earlier, the solid angle model (SAM) is better than the area model, but its inventor, David Singmaster, was disturbed that the theory didn't do better for squatty and tall prisms:

..."I am perplexed that the theoretic results are so far from the observed results and I can see no geometric explanation."

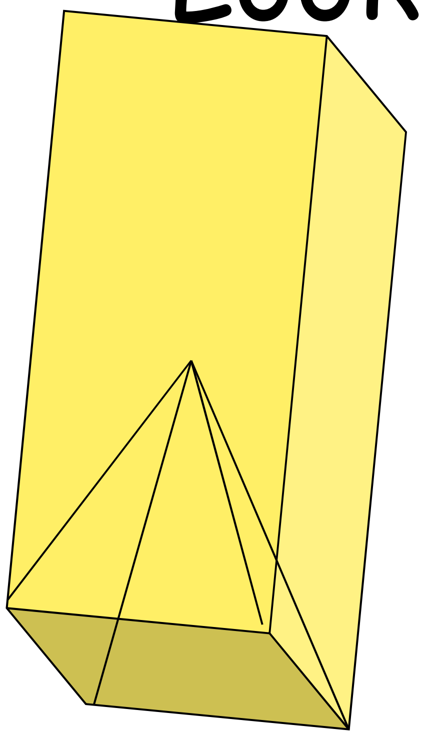


Similarly for cylinders---SAM doesn't do coins or pillars well.



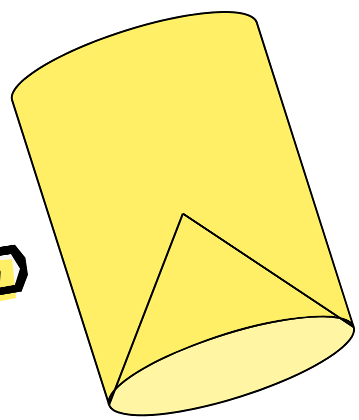
Applied to cylinders, Singmaster's model predicts 8% edge landings when applied to a "10p" coin, surely too large.

Looking closer at SAM



- David Singmaster (1981) appears to have been the first to suggest the solid angle model (SAM)---the solid angle from the center is proportional to the number of landings on that face.

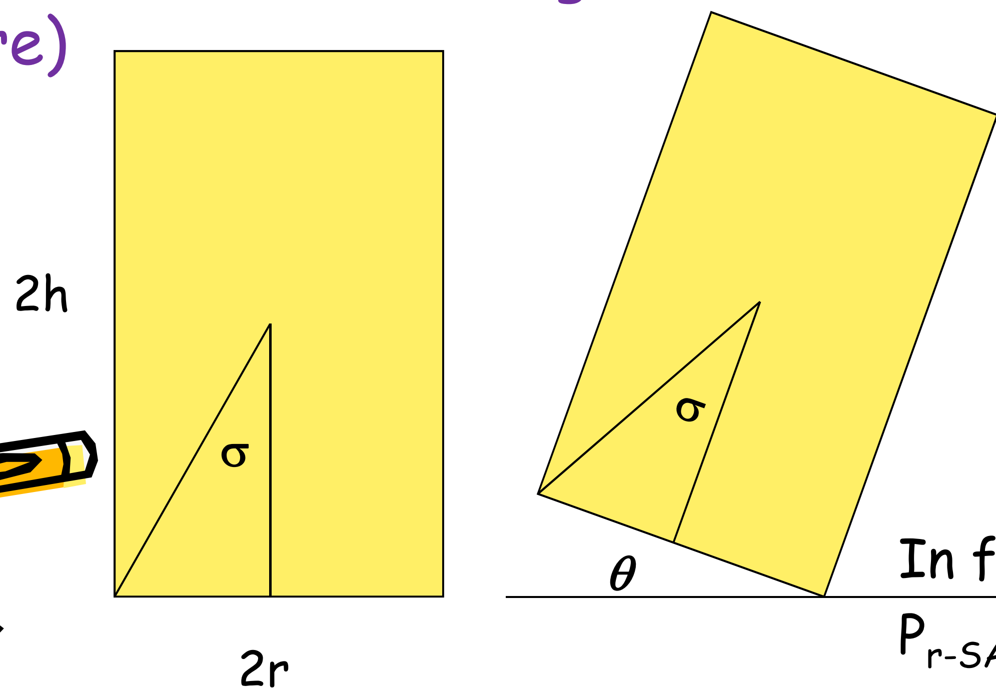
(For boxes, this is exactly what you'd expect if the box were balancing on a corner and compelled to rest on the square below the center of mass. For cylinders, the conical angle to the bottom circle measures the probability that it will land on that face.)

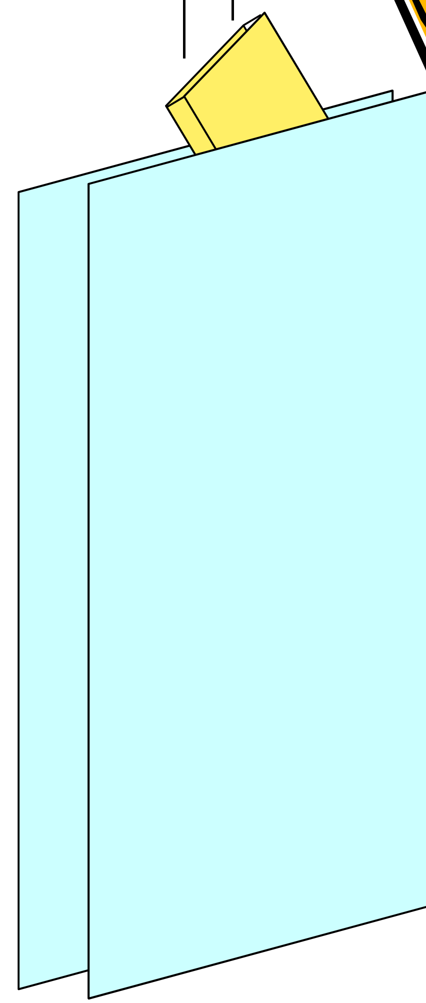


To better understand why SAM has trouble with coins and pillars we decided to look something simpler…

Imagine SAM in 2-D!

- Drop rectangular tiles between two closely-spaced, fixed vertical walls.
- Record whether it lands on r-side or h-side.
- In 2-D SAM, σ is proportional to the number of r-side landings (see figure)



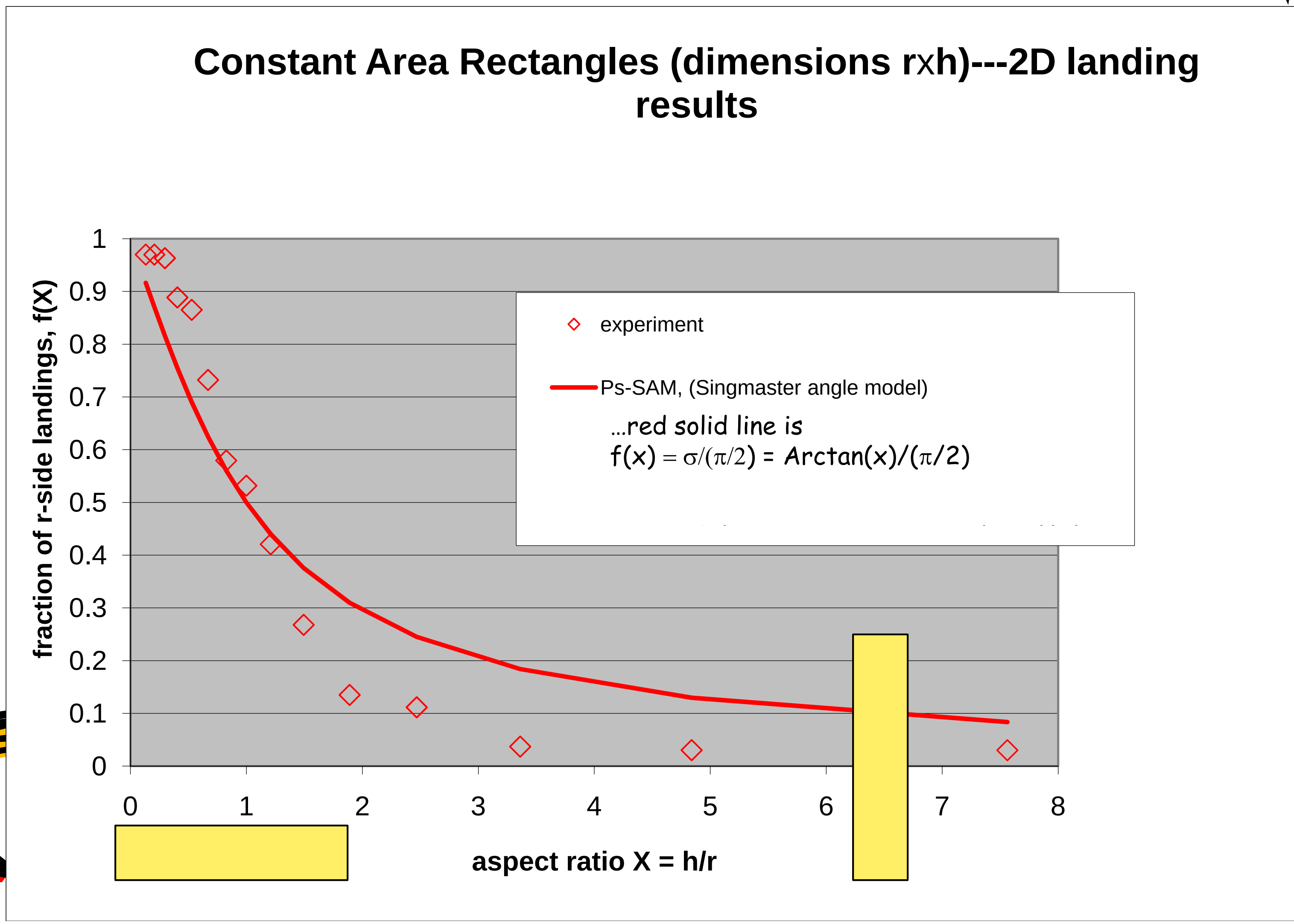


In fact, since $\text{Tan } \sigma = r/h$,
$P_{r\text{-}SAM} = \text{Arctan}(r/h)/(\pi/2)$

So, we cut some equal area rectangles from wood, allowed them to fall ~vertically in a plane & took some data in 2-D;

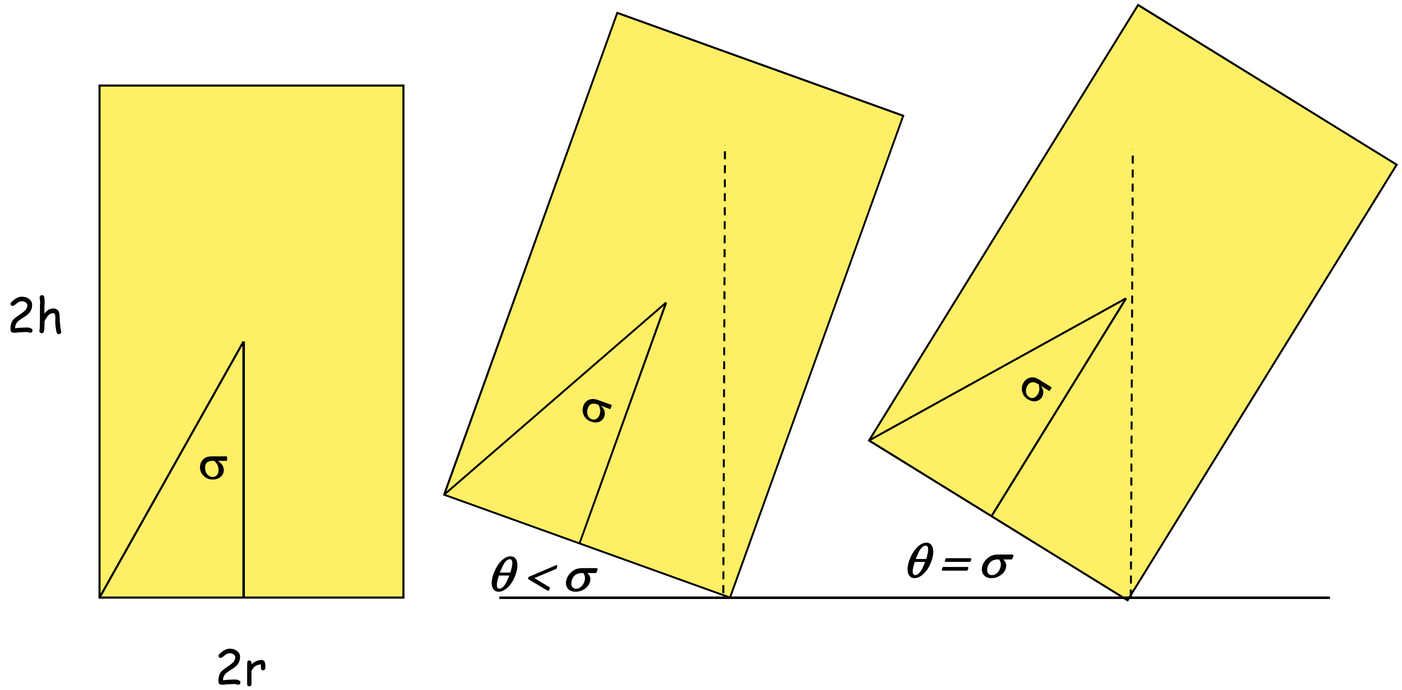
2-D results are similar to 3-D,

(SAM misses the mark for skinny rectangles)



Can we modify SAM?

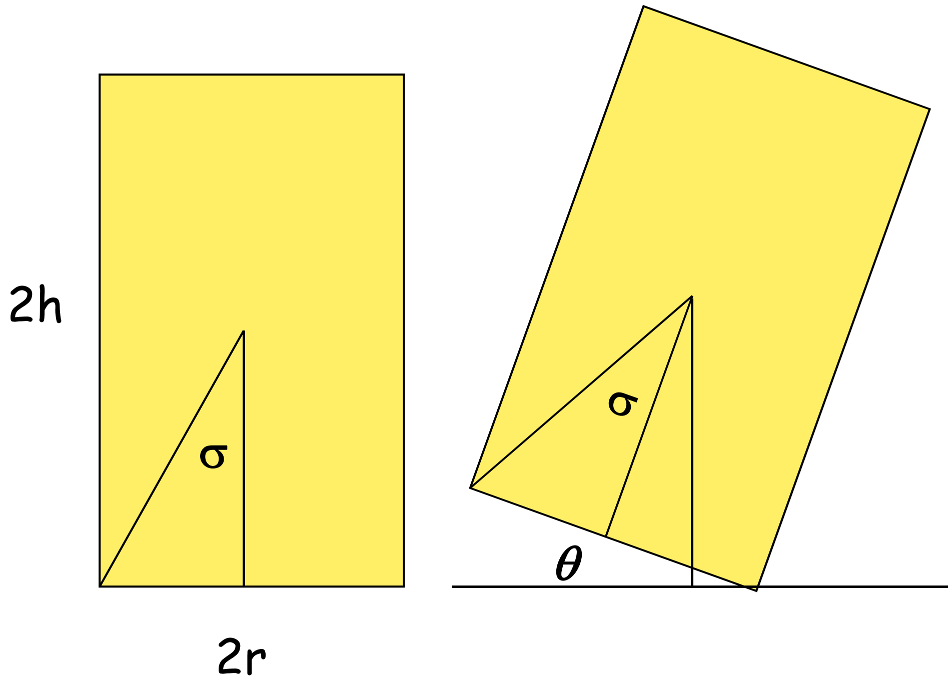
(what about a Boltzmann factor to damp unlikely landings?)



Why? In an ensemble of settling rectangles, the orientations of the rectangle which have the c.m. higher are less likely to be present than those with lower c.m. How much less likely?

Let's try a Boltzmann factor of exp(-constant*E) or since KE is very low, let's weight each possible orientation with a factor of $\exp(-\beta y_{cm})$...

- So, instead of simply $P_r = \int_0^{\sigma} d\theta \Big/ \int_0^{\pi/2} d\theta$, or P_r = Arctan(r/h)/(π/2), we get

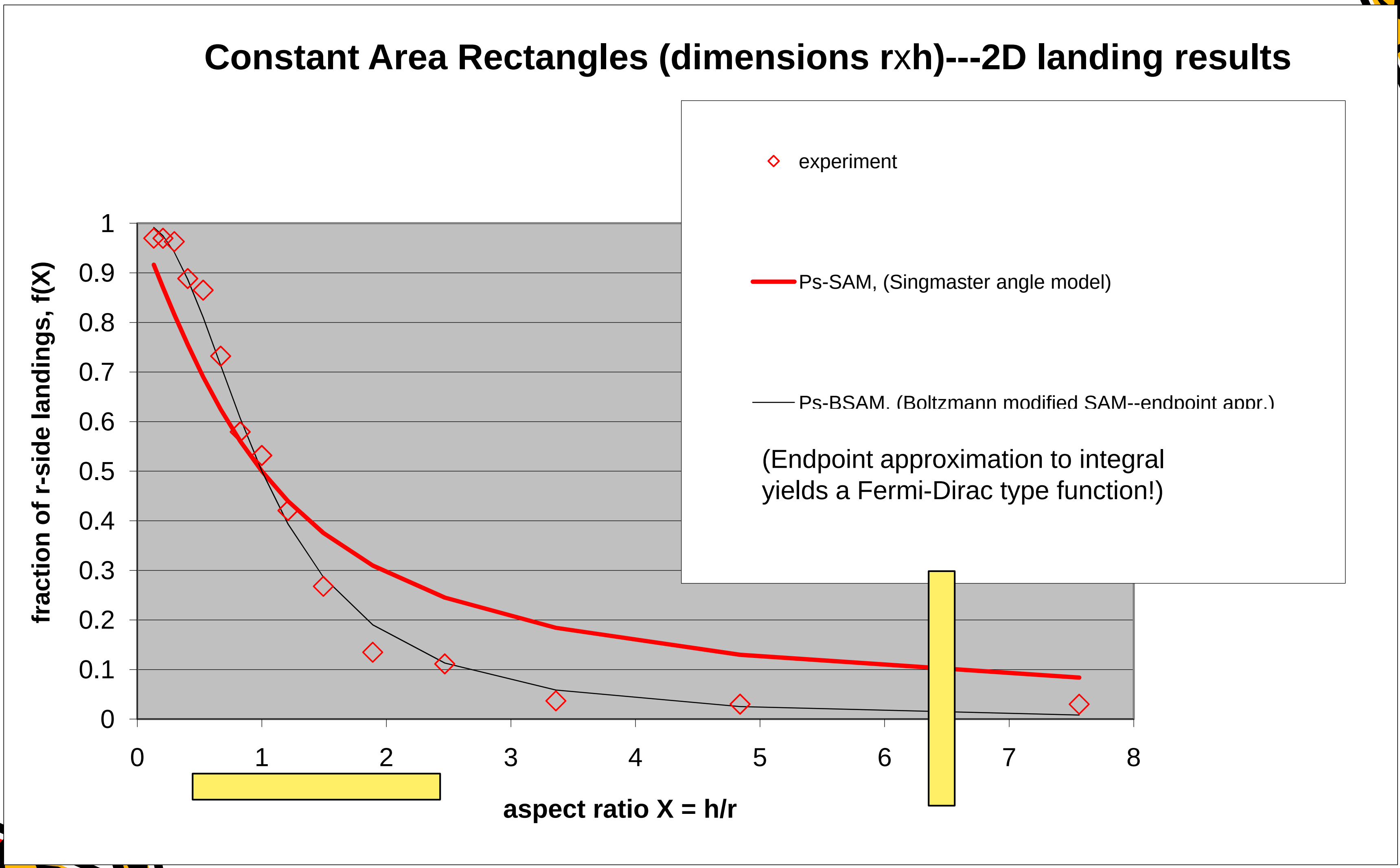


$$P_r = \int_0^{\sigma} e^{-\beta y_{cm}} d\theta \Big/ \int_0^{\pi/2} e^{-\beta y_{cm}} d\theta$$

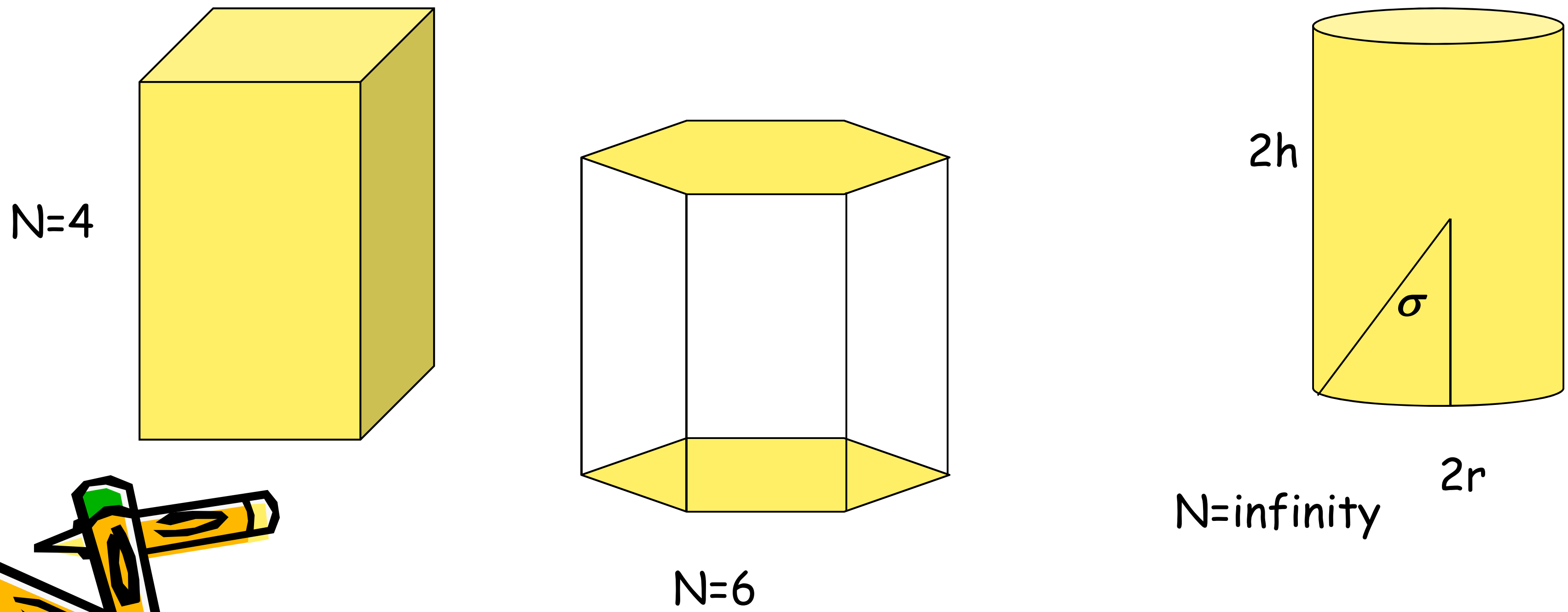
y_{cm}= [r*sin(θ) + h*cos(θ)]/L_{ch}

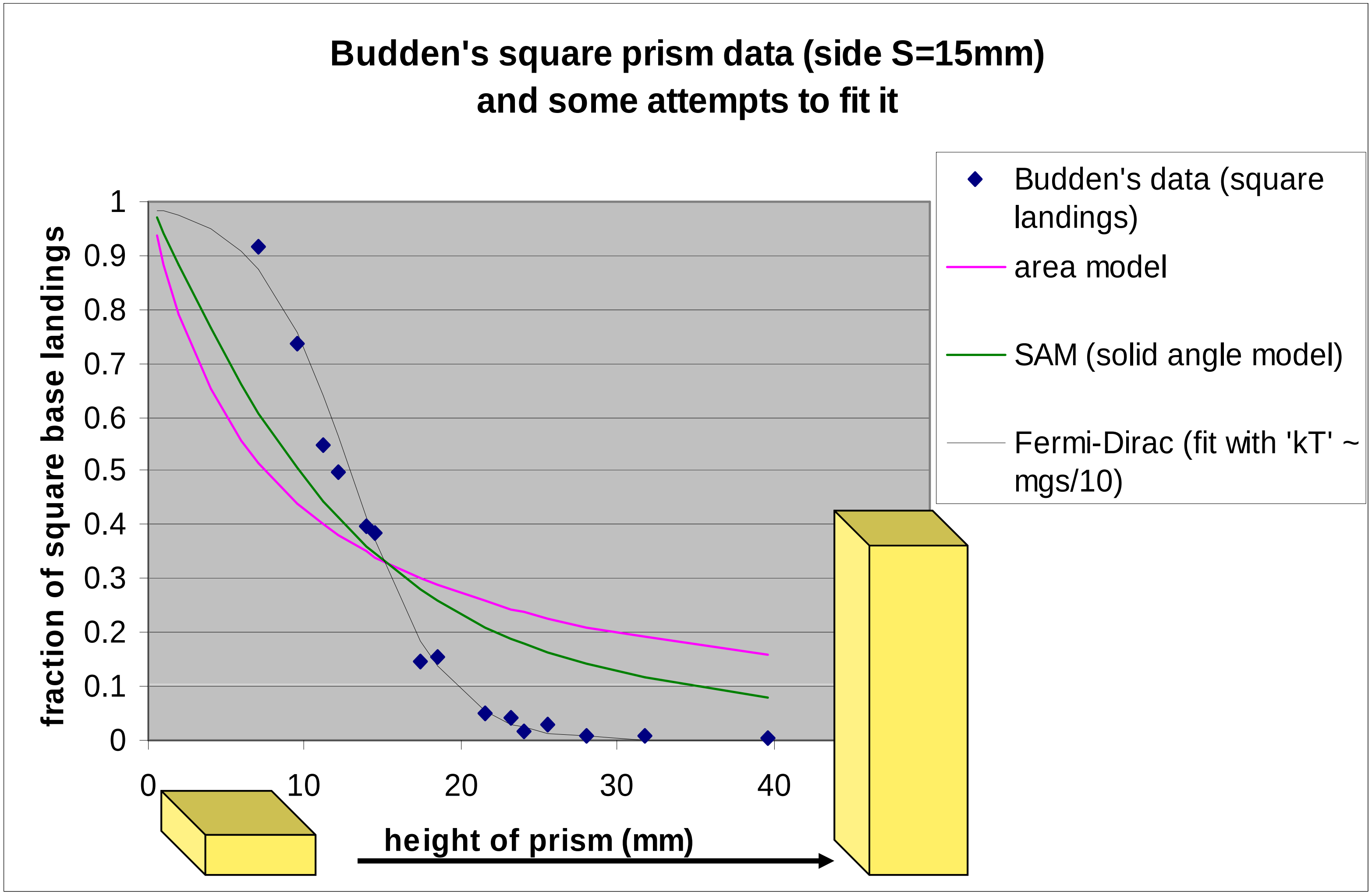
Notice that this modification will reduce the number of high c.m. landings, and will increase the number of low c.m. landings

Results? …not too shabby



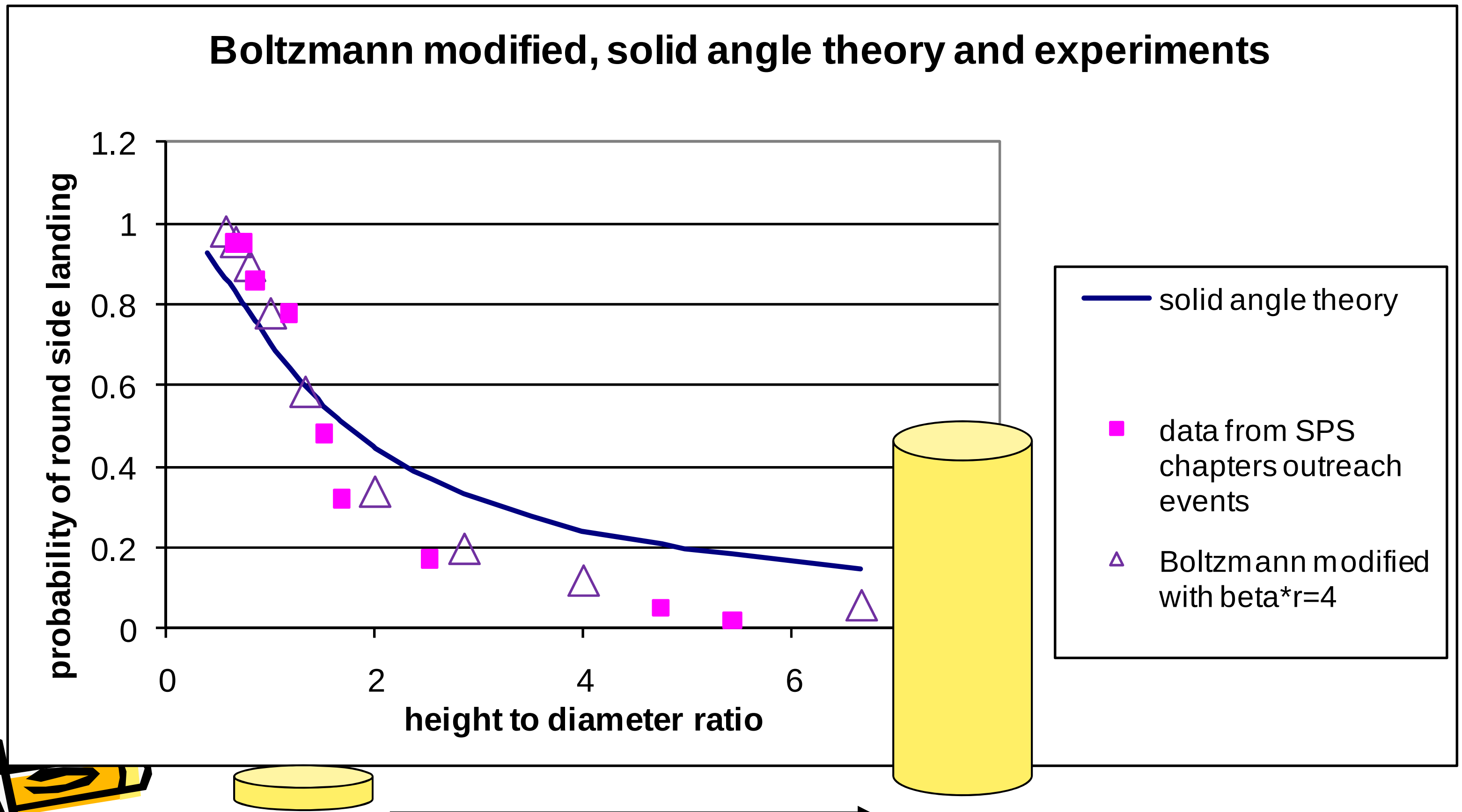
Now move Boltzmann modification to 3-D; consider N-prisms so we can do boxes (N=4) and cylinders (N → infinity)





Works OK for boxes also, as we have seen

Cylinder results are encouraging, also.



Summary and future

- Our SAM-Boltzmann model seems to describe a variety of dropping data pretty well, giving some credence to the underlying assumptions, and some understanding of the curious appearance of a Fermi-Dirac-type function in a classical setting.

- Dropping experiments are good ways to introduce students to probability and distributions.

  - Looking to better understand "temperature" of droppings and to test robustness of SAM-Boltzmann model to other shapes, including those where the center of mass is shifted from the geometric center.

Thanks to

SPS chapters at

- Christian Brothers University,
- Rollins College,
- University of Louisville,
- Carthage College,
- Embry-Riddle Aeronautical University,
- Lafayette College,
- University of North Carolina at Asheville,
- Massachusetts College of Liberal Arts,

ALSO

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- UMd MRSEC students and parents,
- Thomas Olsen, Kendra Rand, Elizabeth Hook, SPS staff and
- Six Flags Physics Day participants

SPS interns, especially

Matt Shanks

Heather Lunn

Morgan Halfhill

Rebecca Keith

Mika McKinnon

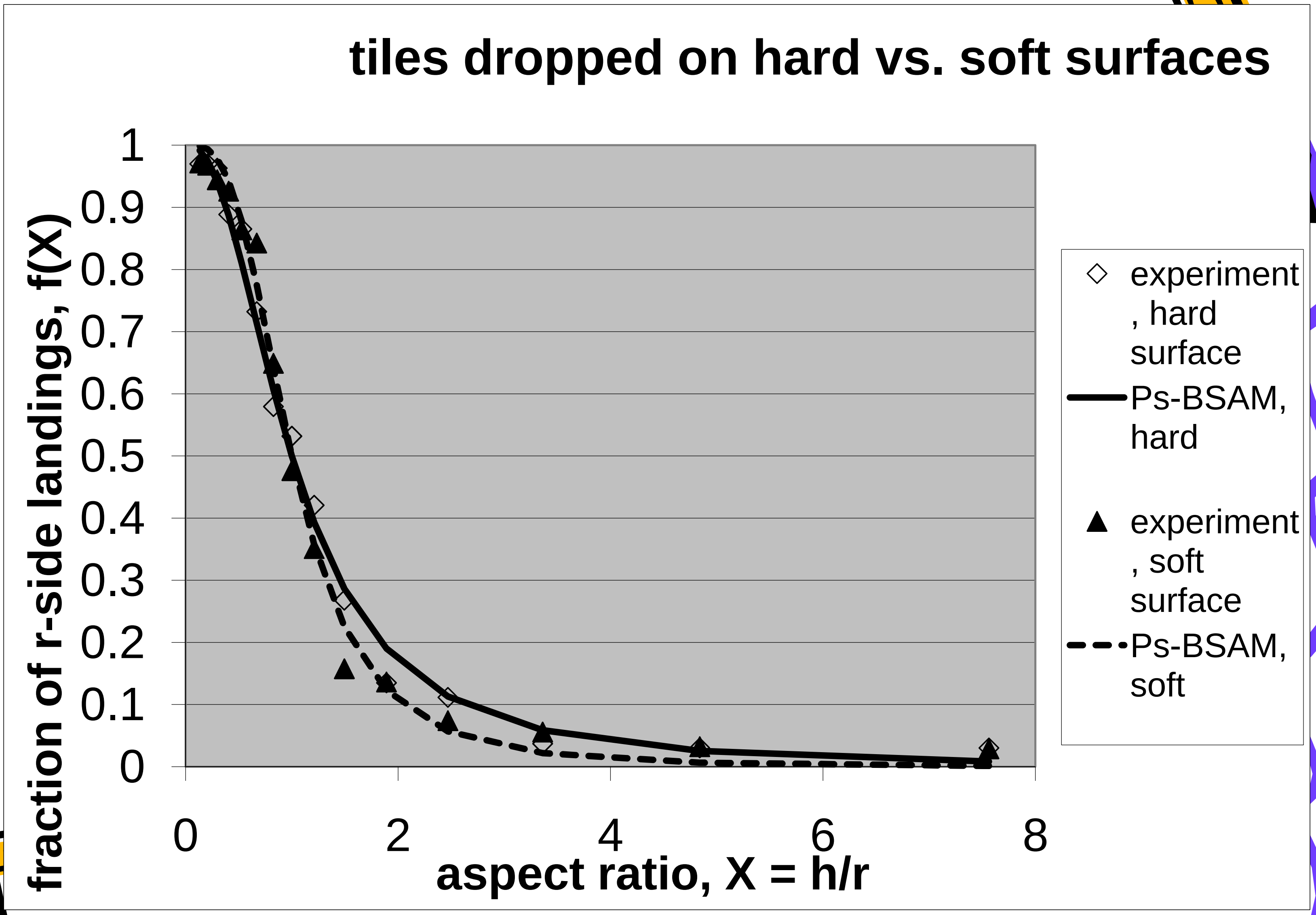
And

Phoebe White,

and Susan White

And you! …any questions?

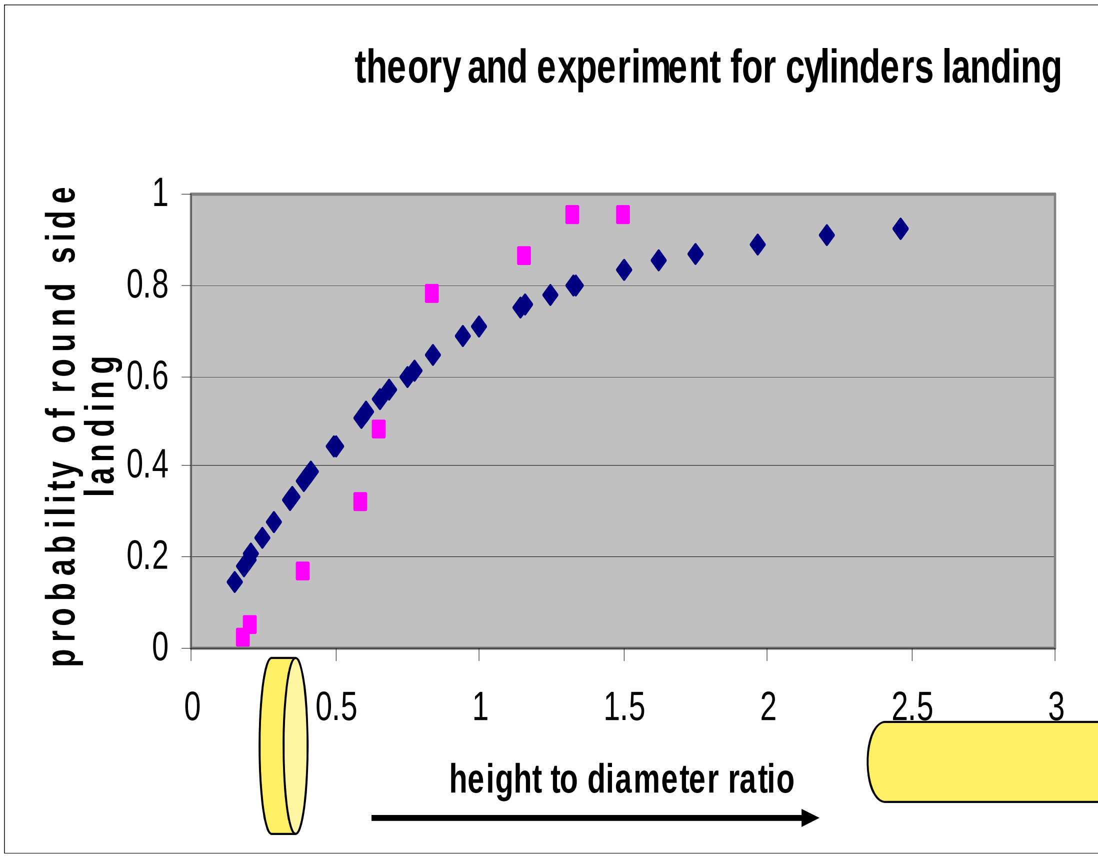
More 2-D data



tiles dropped on hard vs. soft surfaces
fraction of r-side landings, f(X)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
aspect ratio, X = h/r
experiment, hard surface
Ps-BSAM, hard
experiment, soft surface
Ps-BSAM, soft

How does the solid angle theory do for cylinders?

But notice that the solid angle theory predicts too many rolling landings for coins and too few rolling landings for pillars, reproducing Singmaster's lament



Let's move to 2-D to see if things are easier

Applied to cylinders, Singmaster's model predicts 8% edge landings when applied to a "10p" coin.

What about friction, bounciness, etc?

- H. Bondi (of cosmology fame) addresses the issue of a cylinder dropping in 1993 from a purely theoretical mechanics perspective. He starts with an inelastic, perfectly rough floor, then a smooth floor…he finds probability of side landing for coin depends dramatically on the height from which it is dropped among other things. For nickels his model gives about 0.6% edge landings when dropped from 10cm or so.

What about other experiments?

- Daniel Murray and Scott Teare, also in 1993, simulate the bouncing of a dropped cylinder on a frictionless surface allowing various coefficients of restitution, using experiments with hex nuts to help determine appropriate values for parameters in their model. They predict a nickel will land on its edge 0.017% when dropped from about 15cm.

My prediction for a coin landing on its edge?

- Hmmm…on a hard surface or soft?
- From 15cm or 2 meters?
- Cylinders, "10p coins" or actual nickels (with their beveled edges)?
- I'm pretty sure it's between 0% and 8%

(…more details to come in future work)