

# Characterization of Large-scale Velocity Fluctuations in the Princeton MRI Experiment

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## Introduction

### Accretion Disks

- Accretion disks are disks of gas, dust, and plasma surrounding massive point-like objects such as black holes or stars.
- The rate of accretion onto the central objects is limited by how quickly angular momentum can be transported outwards through the disk; observed rates of accretion are much larger than can be explained by collisional viscosity alone.
- To explain the enhanced transport, accretion disks are assumed to be turbulent with one of the proposed mechanisms for disk turbulence being the Magneto-Rotational Instability (MRI).

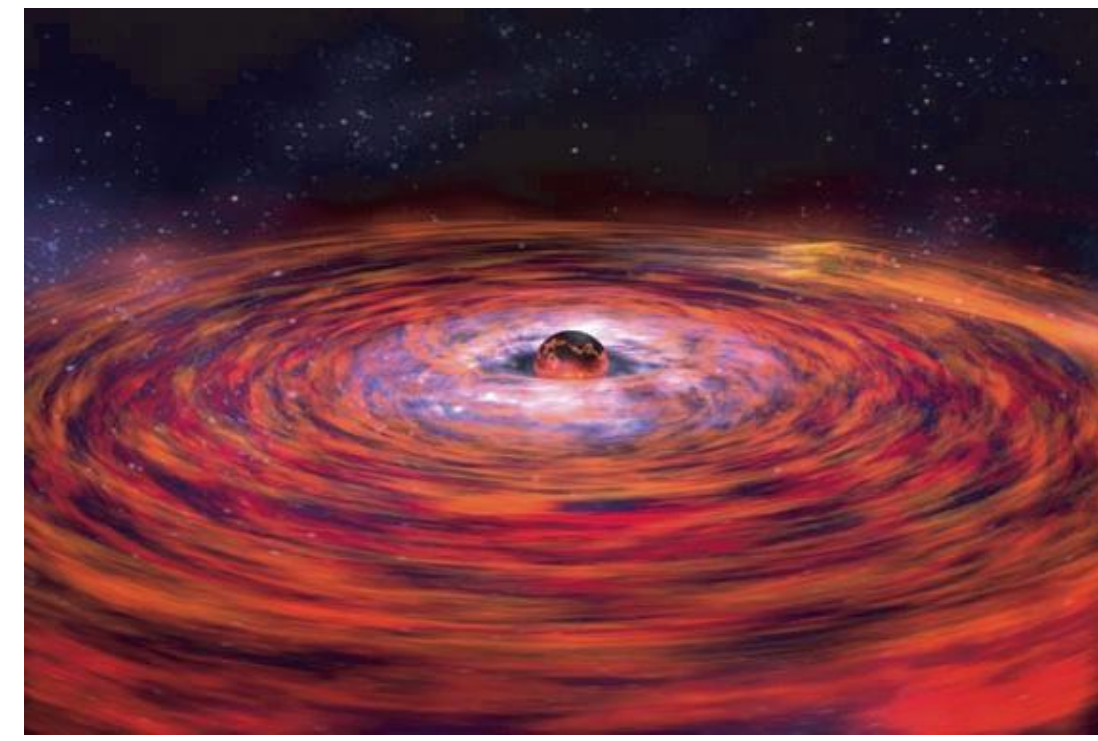


Figure 1. A computer generated graphic of an accretion disk around a compact stellar object.

### MRI Experiment

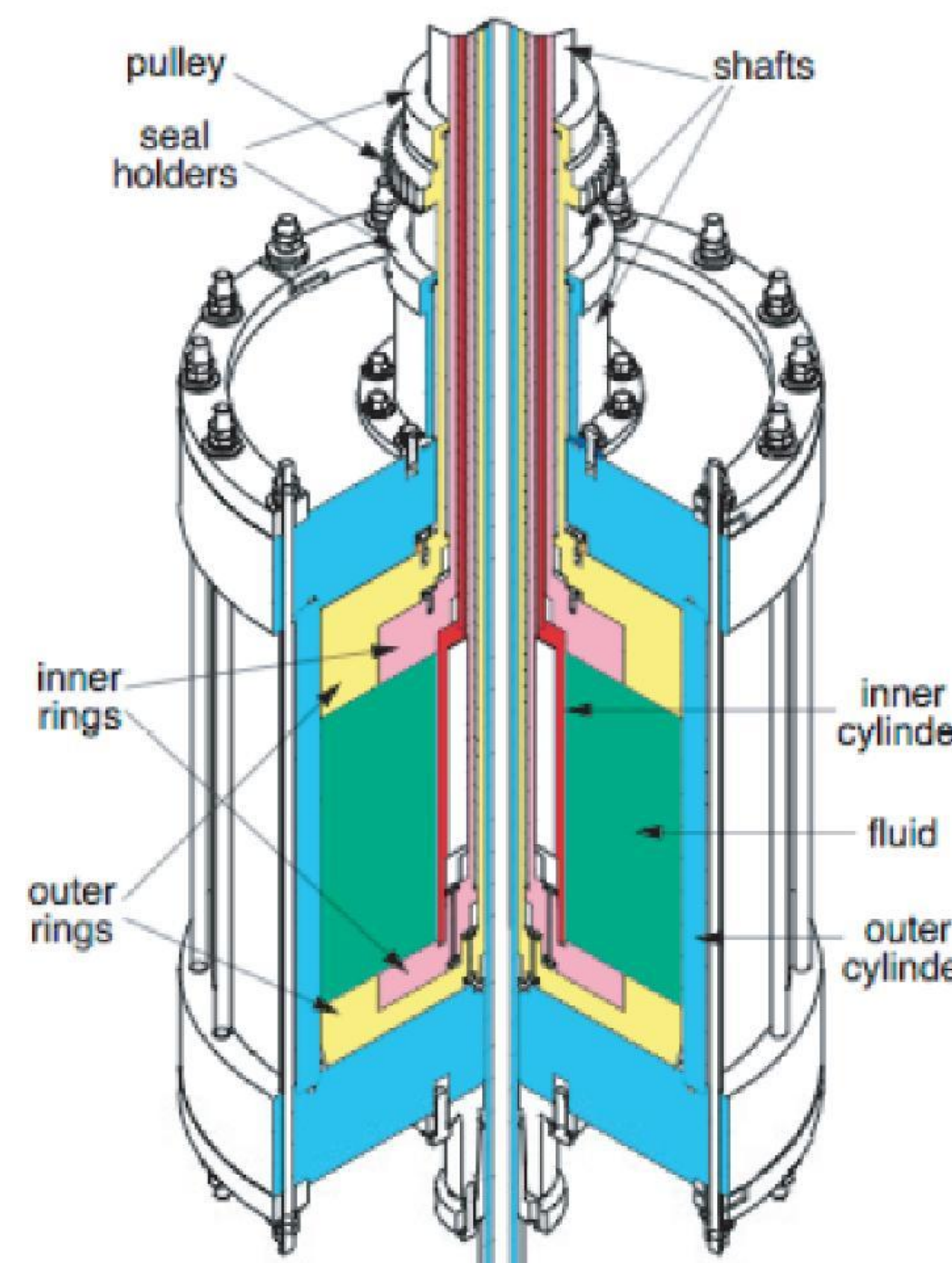


Figure 2. A cross sectional diagram of the MRI experiment.

- The Princeton MRI Experiment is a wide-aspect ratio Taylor-Couette experiment.
- Two independently rotating endcap rings suppress secondary circulation.
- The working fluid is GaInSn
- Vertical magnetic field up to 5000 Gauss can be applied.

### Ultrasound Diagnostics

- Ultrasonic Doppler velocimetry (UDV) is a method of measuring the velocity flows in fluid by have a transducer emit ultrasonic pulses and then detecting the ultrasonic echoes reflected by microparticles contained in the liquid. Measuring the Doppler shift from the reflected pulses allows the velocity of the particles to be calculated.
- The UDV probes in the MRI experiment measure the velocity field at the midplane, with one radial chord probe, two azimuthal chord probes and one vertically oriented probe that can be moved to different radii.

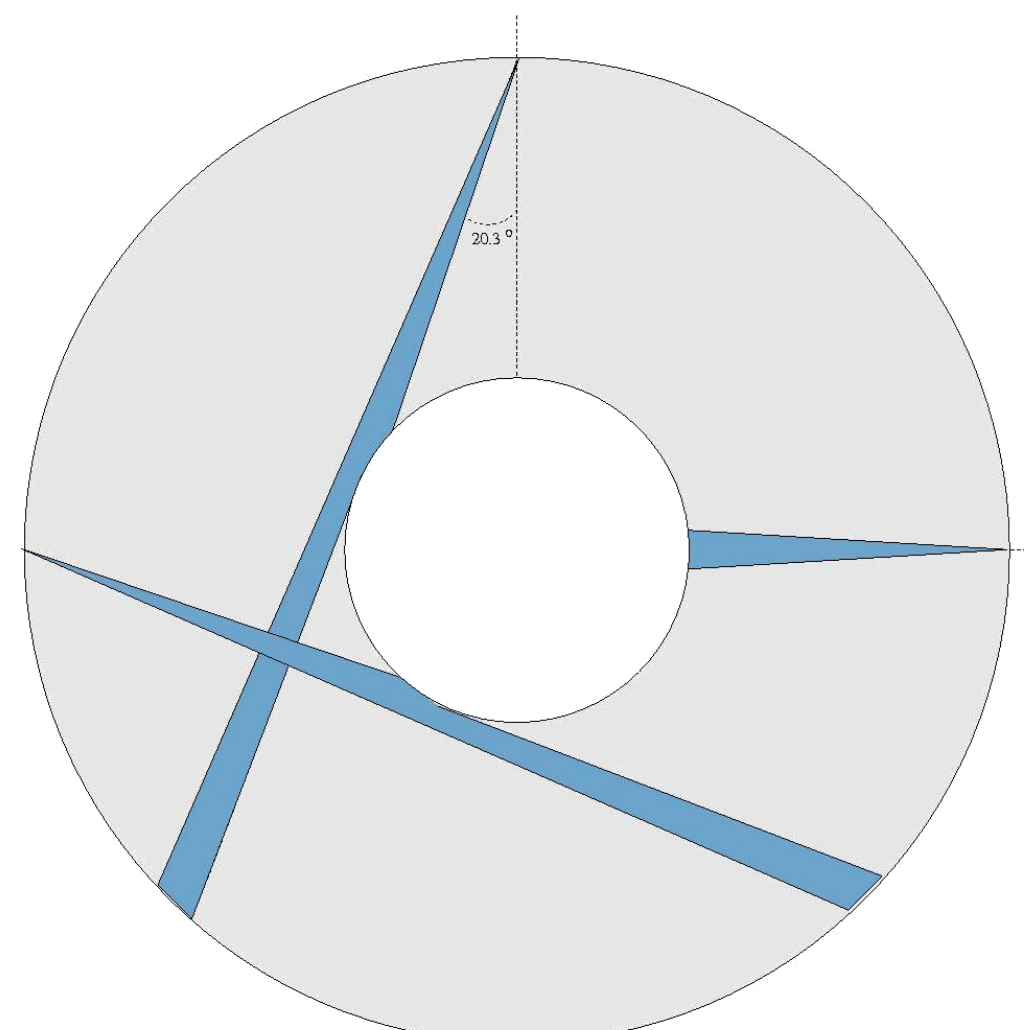


Figure 3. A view of the UDV probes from the vertical axis.

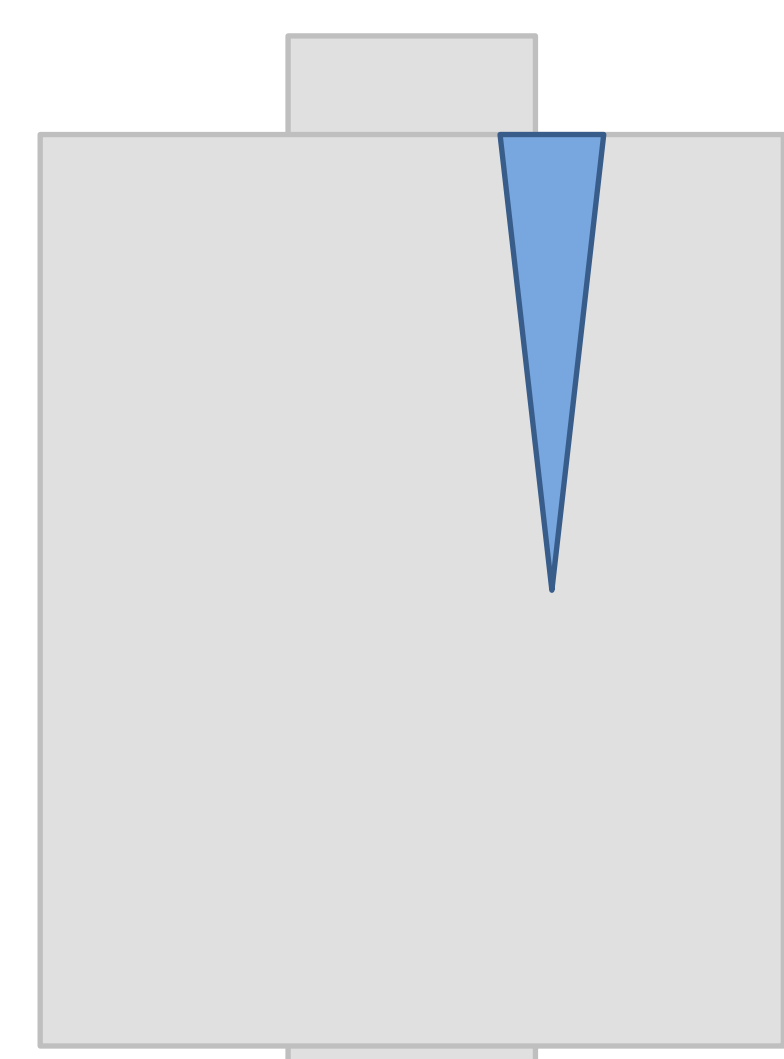


Figure 4. A side view of the vertical UDV probe which is directed upwards.

## Purpose

- When the applied magnetic field is strong enough and there is sufficient flow shear, large oscillations are observed in the azimuthal flow that occur below the threshold for MRI.
- These oscillations are m=2 in the linear phase, but become m=1 in the saturated state.
- Previous work has extensively observed these oscillations, but did not characterize them, which is the goal of this work.

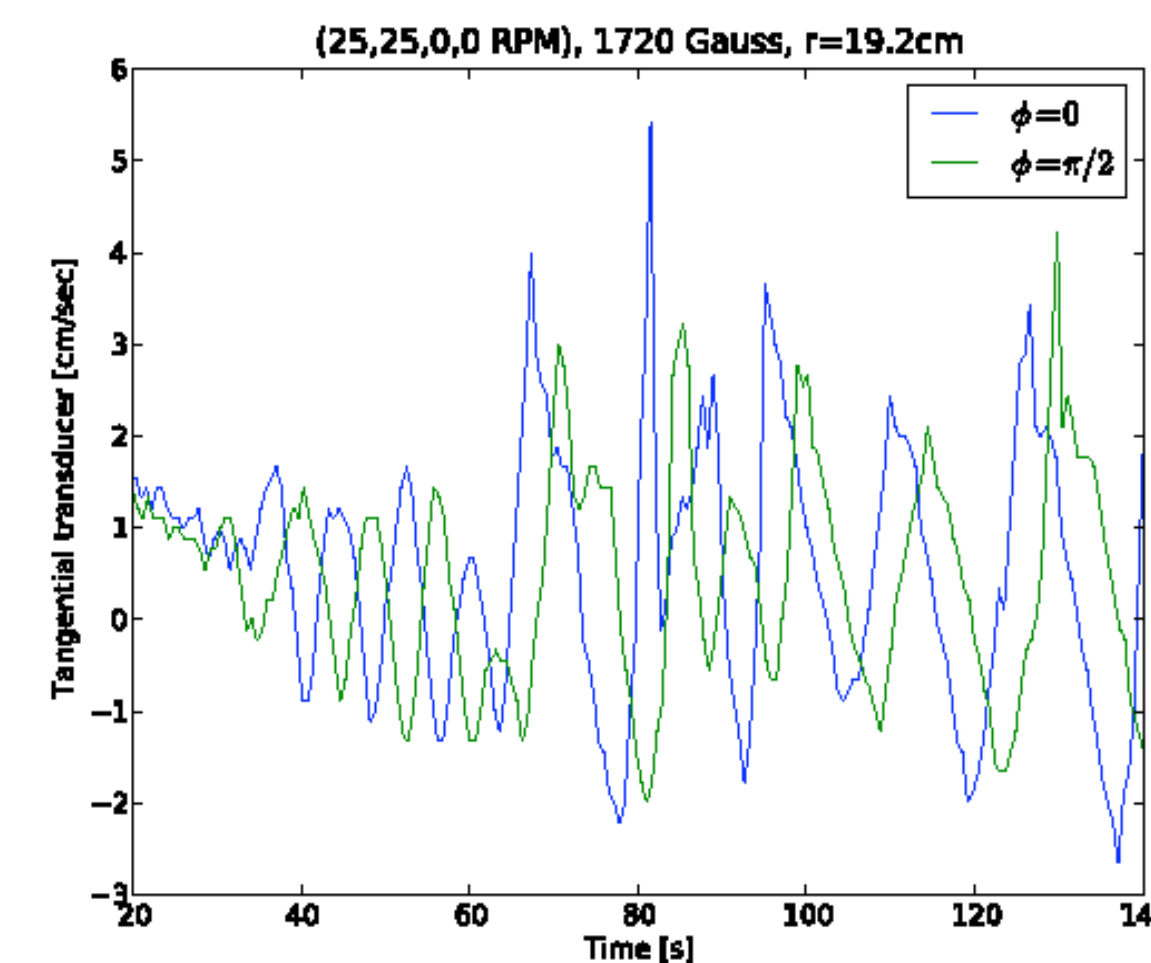


Figure 5. A plot of the two azimuthal probes showing the oscillations

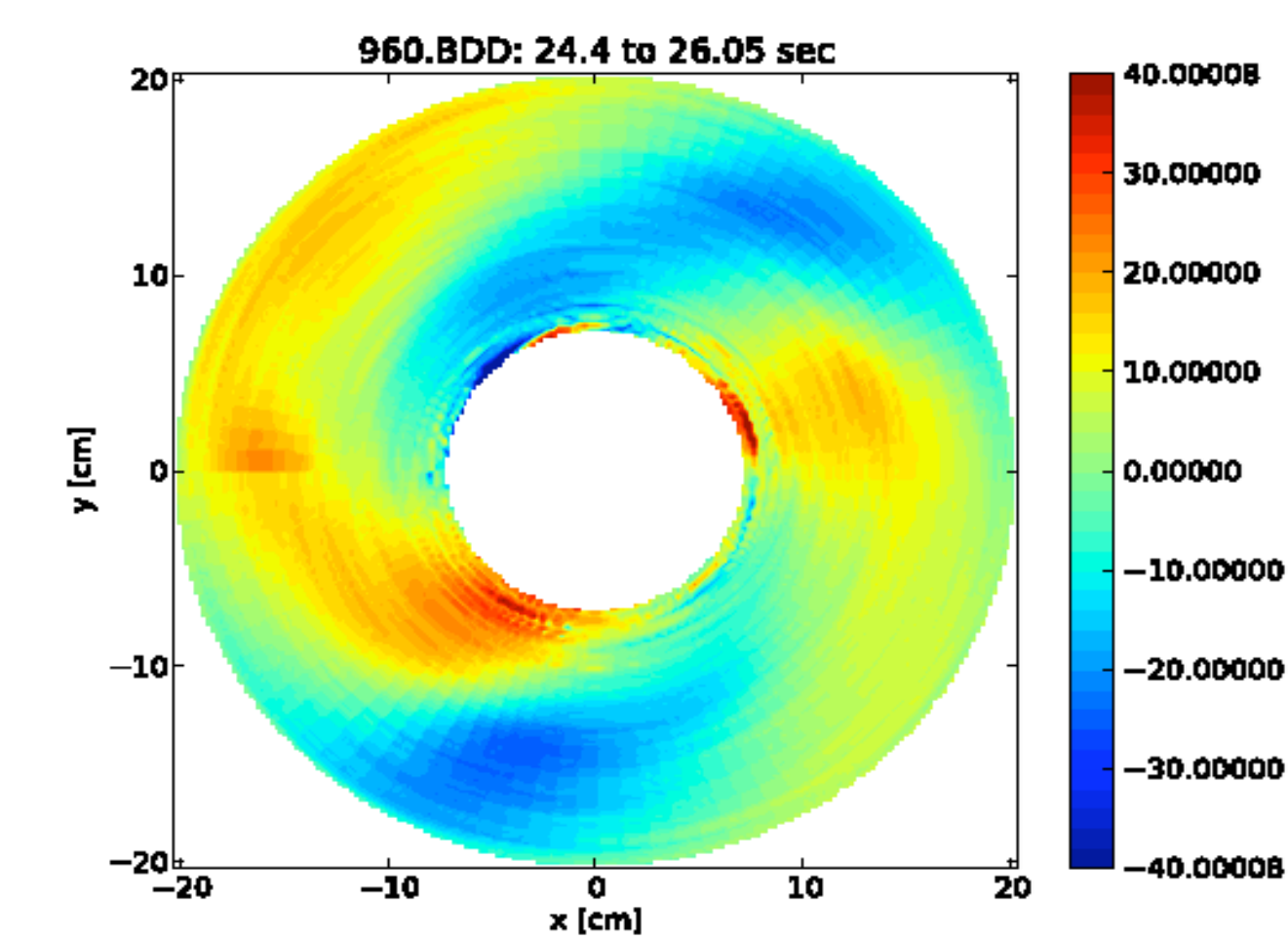


Figure 6. A contour plot graphically showing the azimuthal velocities

## Methods

### Modeling

- In order to characterize the oscillations, contour plots of the actual azimuthal and radial velocities were made to observe the appearance of the velocity structures as shown in figures 7 and 9.
- Equations were created for  $v_\theta$  and  $v_r$  that accurately reproduced the large scale structures and magnitudes that were seen in the data for both the m=1 and m=2 oscillation modes as shown in figures 8 and 10

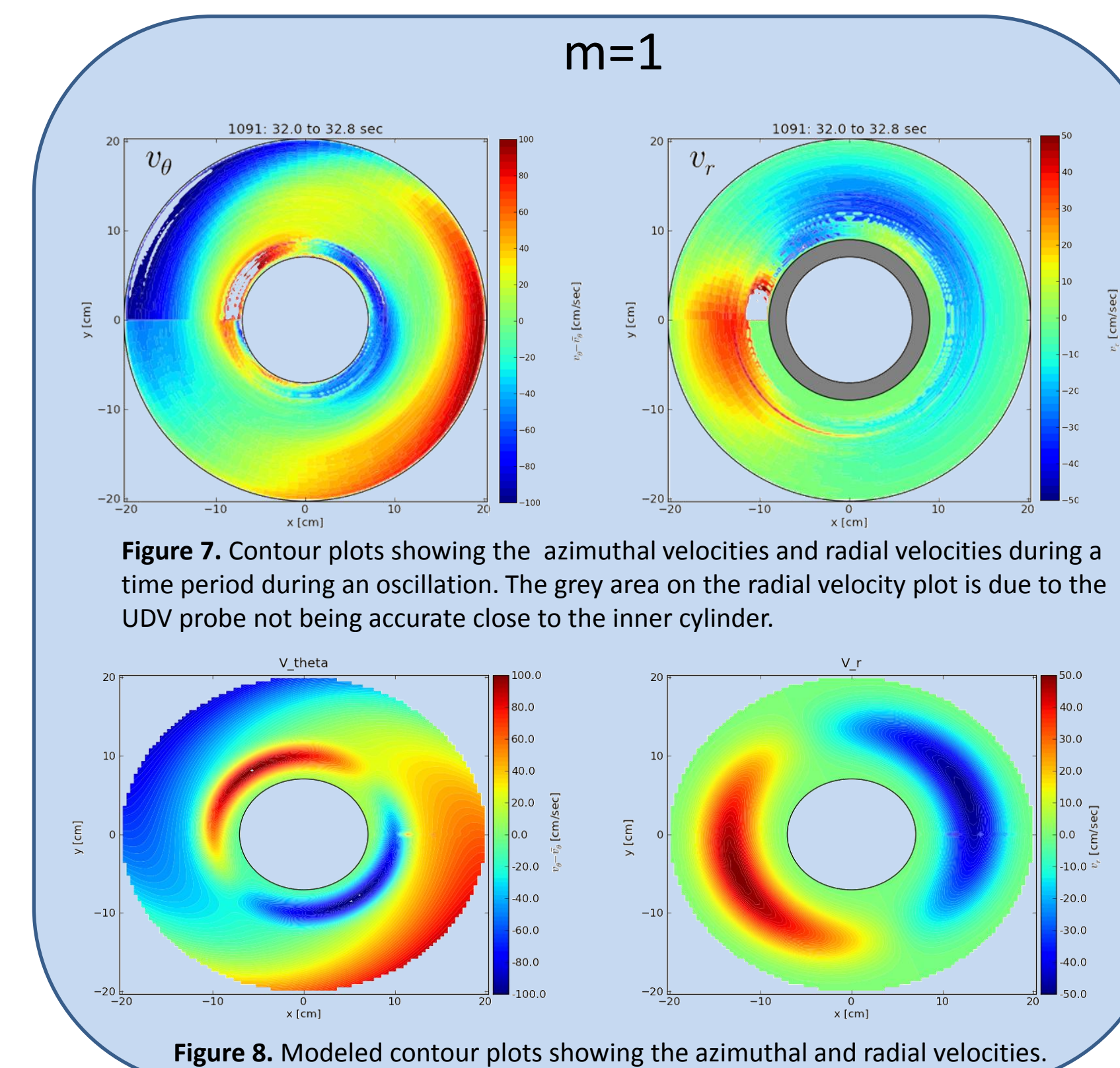


Figure 7. Contour plots showing the azimuthal velocities and radial velocities during a time period during an oscillation. The grey area on the radial velocity plot is due to the UDV probe not being accurate close to the inner cylinder.

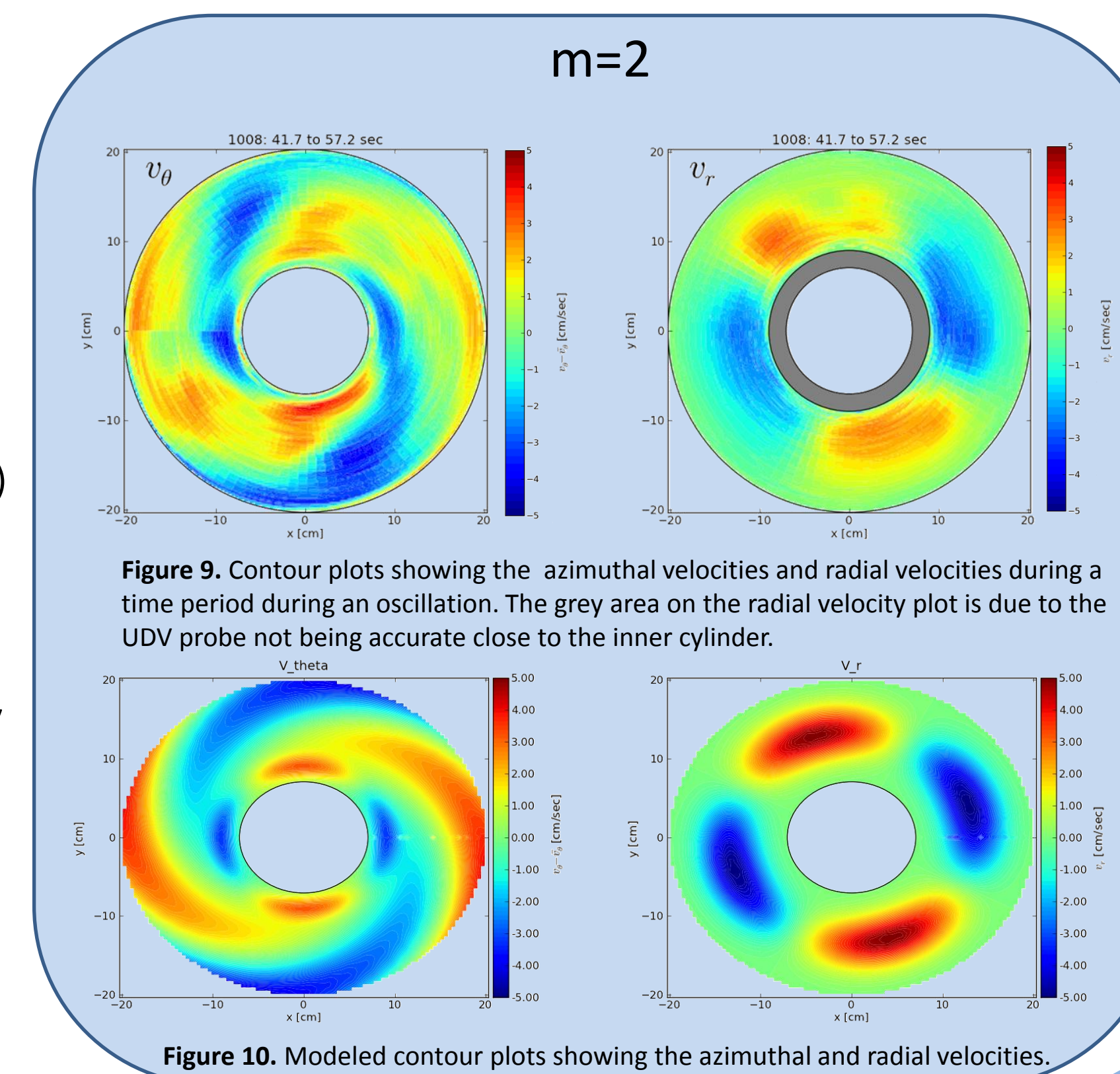


Figure 8. Modeled contour plots showing the azimuthal and radial velocities.

### Calculating $\frac{\partial v_z}{\partial z}$

- Since the MRI experiment is a closed system it must satisfy the cylindrical continuity equations which is as follows:  

$$\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \quad (1)$$
- From the continuity equation and the models for  $v_\theta$  and  $v_r$  the  $\frac{\partial v_z}{\partial z}$  term can be calculated to see if it correctly predicts what the vertical velocity flow structure is.

## Results

- Solving equation (1) yields  $\frac{\partial v_z}{\partial z}$  which can then be multiplied by a height on the cylinder to get units of velocity.

- The derived  $v_z$  flows should oscillate with  $v_r$  and  $v_\theta$ .

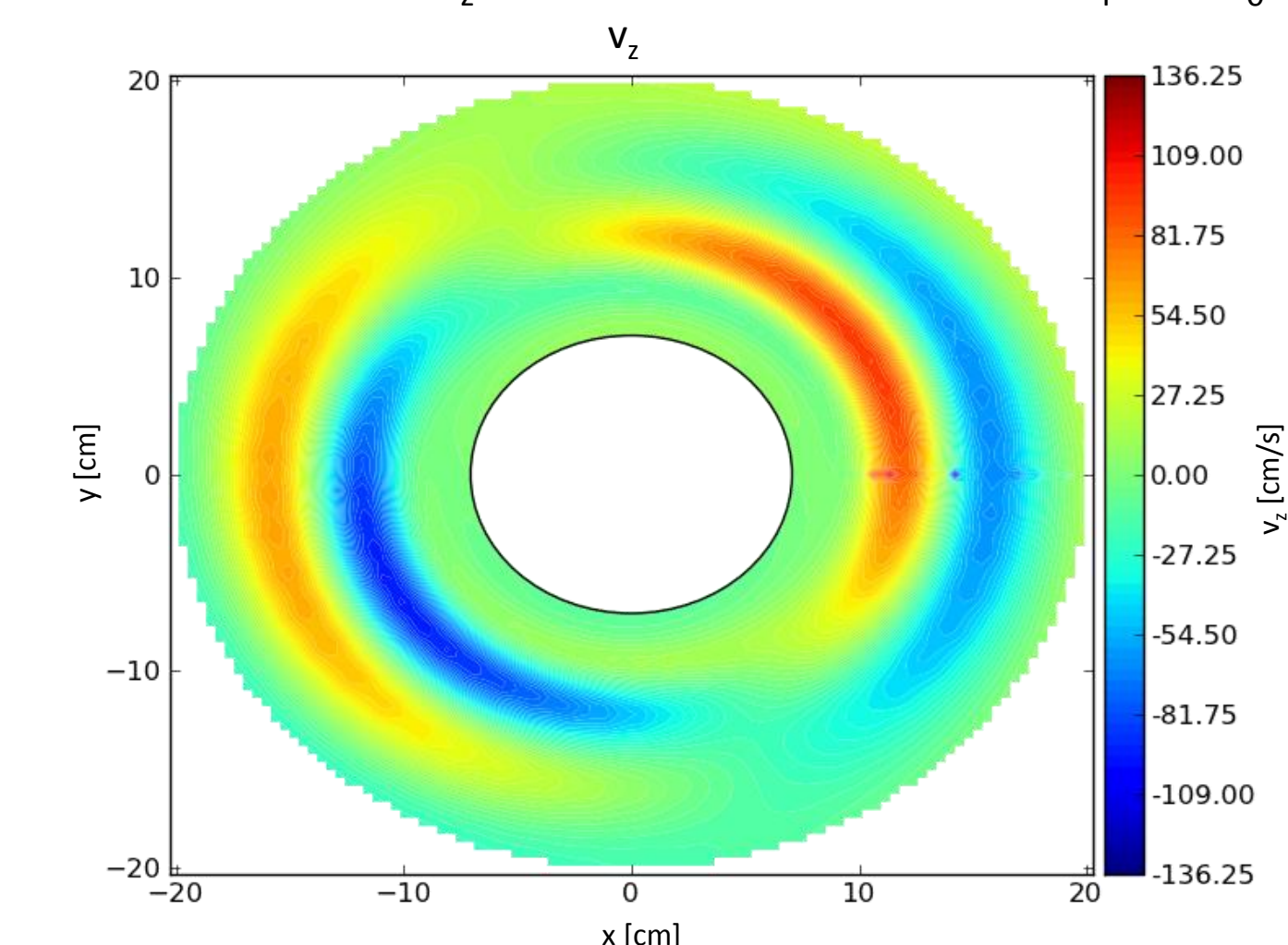


Figure 11. Contour plot of the  $\frac{\partial v_z}{\partial z}$  term in the model of the m=1 oscillation multiplied by one fourth of the cylinder height.

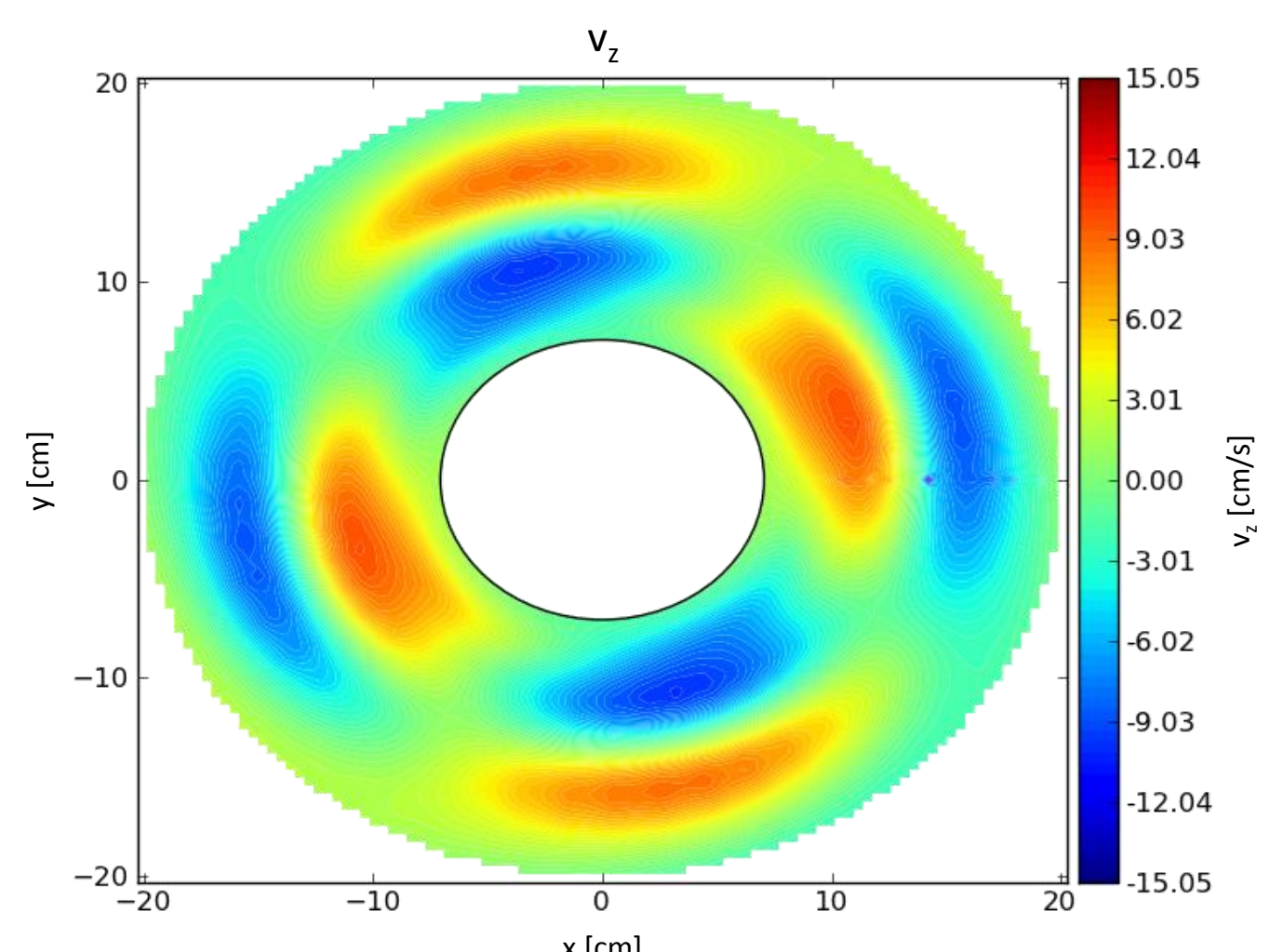


Figure 12. Contour plot of the  $\frac{\partial v_z}{\partial z}$  term in the model of the m=2 oscillation multiplied by one fourth of the cylinder height.

- The below figures show two different methods of measuring the vertical velocities; figure 12 with two azimuthal probes at different vertical heights and figure 13 with a direct measurement of the vertical velocity

- Both measurements suggest the  $v_z$  is not dynamically significant.

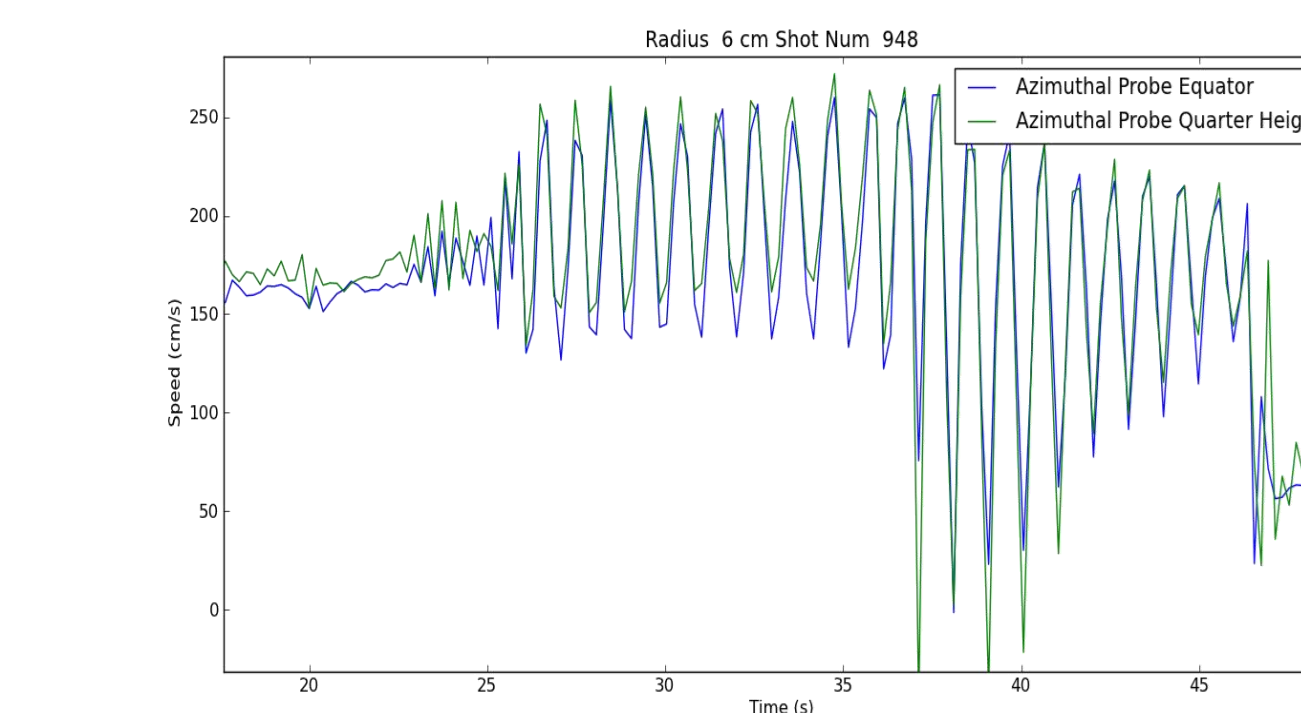


Figure 13. Plot showing oscillations when the two azimuthal UDV probes that at different heights were installed.

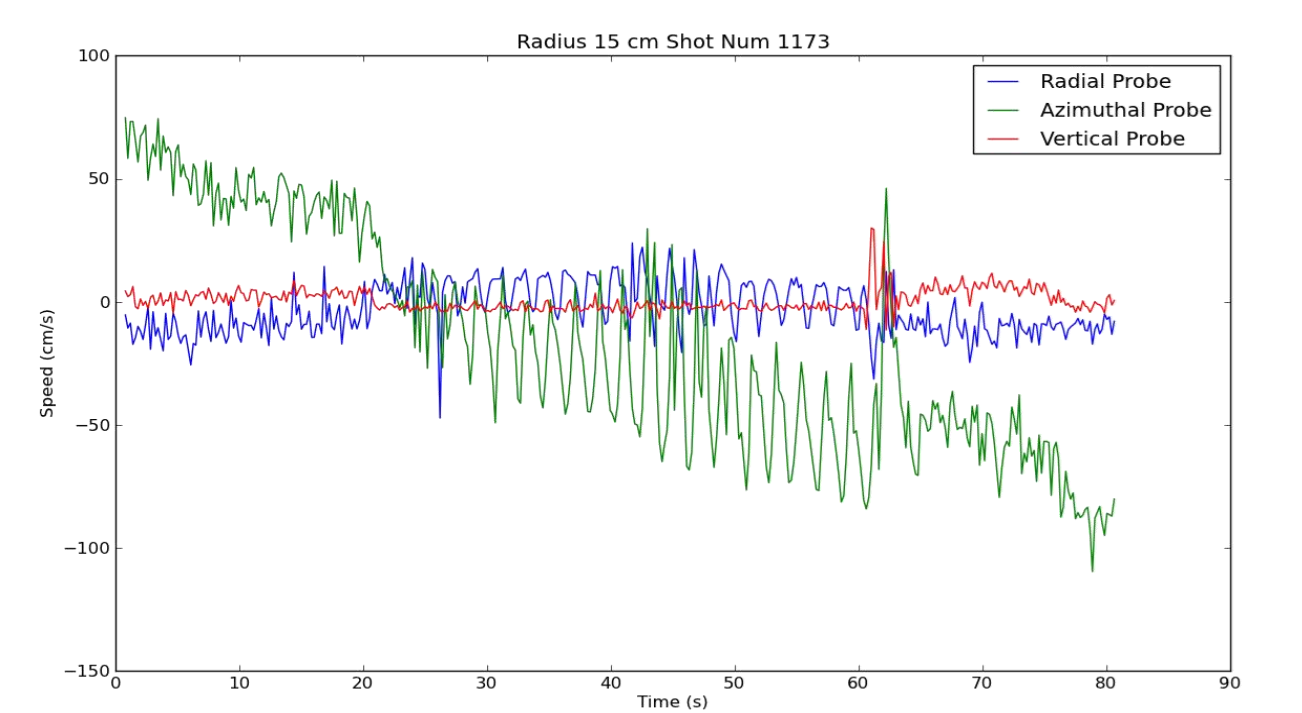


Figure 14. Plot showing oscillations when the vertical UDV probe was installed.

### Current Density Results

- Current density was calculated from Ohm's law (2) and the constraint that the current is divergence free (3).

$$\mathbf{J} = \sigma(-\nabla\Phi + \mathbf{v} \times \mathbf{B}) \quad (2)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (3)$$

- The potential  $\Phi$  was calculated by solving the following differential equation numerically

$$\nabla^2 \Phi = \nabla \cdot (\mathbf{v} \times \mathbf{B}) \quad (4)$$

- In order for this calculation to be possible, it was assumed that  $\Phi$  is constant in the z direction.

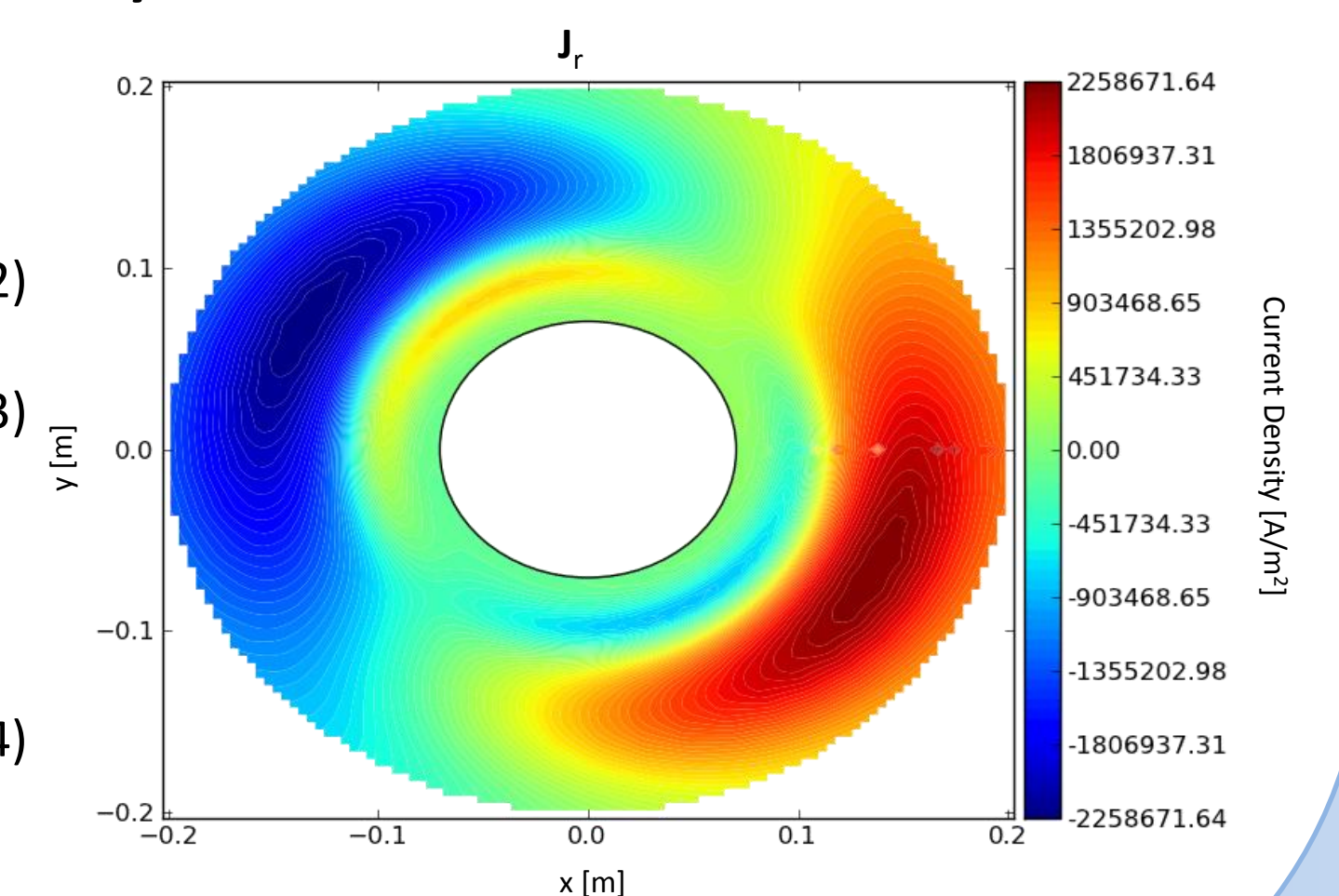


Figure 15. Contour plot of the radial component of the calculated current density from the model for m=1.

## Conclusions

- Model fits to the experimental data show that continuity is far from satisfied with only  $v_r$  and  $v_\theta$ , however this does not agree with measurements of  $v_z$  which show no oscillations.
- Calculated current density proved to be extremely large, it is possible that the assumption of  $\Phi$  being constant in the z direction suggests that return currents through the boundary layer are important.
- Future work could include installing probes to measure the z component of the potential in order to correctly calculate the current densities.

## Acknowledgments

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