From Red Cells to Skiing to a New Concept for a Train Track

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Collaborators

**Red Cells**
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**Mechano-transduction**
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Steve Cowin, City College of New York
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**Skiing and Train track**
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Yiannis Andreopoulos, City College of New York
Red and White Cell Motion in Capillaries

Vink and Duling (1996)
Sliding Motion of a Membrane Over a Thin Surface Glycocalyx
Feng and Weinbaum, JFM 422: 281 (2000)

\( h_2 \) is fixed in the model.

\( h_1 \) changes. \( k = \frac{h_2}{h_1} \)

\[ \alpha = \frac{h_2}{\sqrt{K_p}} \]
Two-Dimensional Lubrication Theory for the Brinkman Medium

Brinkman equation:

\[ \nabla p = \mu \left[ \nabla^2 - \frac{1}{K_p} \right] V \]

Dimensionless Reynolds-Type Equation:

\[
\frac{\partial}{\partial x} \left[ fU + \frac{1}{\alpha^2} \frac{\partial p}{\partial x} (2f - h) \right] + \frac{L^2}{W^2} \frac{\partial}{\partial y} \left[ fU + \frac{1}{\alpha^2} \frac{\partial p}{\partial y} (2f - h) \right] \\
= U \frac{\partial h}{\partial x} + V \frac{\partial h}{\partial y} - W,
\]

\[ f = \frac{\cosh \alpha h - 1}{\alpha \sinh \alpha h}, \quad \alpha = \frac{h_2}{\sqrt{K_p}} \]
Pressure Distribution and Equal Pressure Contours Under a Snowboard

\((L/W = 10, h_2 = 2\text{cm}, \alpha(h_2) = 100)\)
Comparison of a Red Cell and SnowBoard
Schematic of Dynamic Snow Compression Apparatus

\[ m \frac{d^2 h}{dt^2} = -mg + F_{\text{air}} + F_{\text{snow}} \]
Comparison Between Theoretical and Experimental Pressure Profiles

![Graph showing comparison between theoretical and experimental pressure profiles.](image)

- $K_{p0} = 2.5 \times 10^{-10} \text{ m}^2$
- $K_{p0} = 1.0 \times 10^{-9} \text{ m}^2$
- $K_{p0} = 7.5 \times 10^{-10} \text{ m}^2$
- $K_{p0} = 5.0 \times 10^{-10} \text{ m}^2$

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Periodic Structure of the Endothelial Glycocalyx

Squire, Chew, Nenji, Neal, Barry and Michel
Hexagonal Array seen near Inner Surface Of Glycocalyx in Freeze-fracture

Squire, Chew, Nenji, Neal, Barry and Michel (2001)
Model for Mechanotransduction
Model for Flow in Capillary

**Governing Equations**

**Core – Navier-Stokes Equation**

\[
\frac{dP}{dZ} = \mu \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U_c}{\partial R} \right)
\]

**Glycocalyx-Brinkman Equation**

\[
\frac{dP}{dZ} = \mu \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U_m}{\partial R} \right) - \frac{\mu}{K_p} U_m
\]

Sangani and Acrivos (1982)

\[
K_p = \frac{-\frac{1}{2} \ln(c) - 0.745 + c - \frac{1}{4} c^2 + O(c^4)}{4c}
\]

c solid fraction
Drag Force Distribution on Each Core Protein

5 µm diameter capillary
Total drag = 1.4 x 10^{-3} pN
Deflection of Core Protein

**Beam equation**

\[
EI \frac{d^4y}{dx^4} = Q(x)
\]

**Loading**

\[
Q(x) = P_m \delta(x - L) + q_m(x)
\]

- **P_m** - Concentrated force on tip
- **q_m** - Distributed force along core protein

**Boundary conditions**

\[
y(0) = 0; \quad \frac{dy}{dx} \bigg|_{x=0} = 0
\]

\[
EI \frac{d^2y}{dx^2} \bigg|_{x=L} = 0; \quad EI \frac{d^3y}{dx^3} \bigg|_{x=L} = P
\]
Relaxation Of Endothelial Surface Layer

Vink, Duling and Spaan (2001)

Characteristic time

\[ T_2^* = 0.38 \text{ s} \]
Flexural Rigidity of Core Protein

Novel Beam Equation:

\[ EI \frac{\partial^4 y}{\partial x^4} = -\frac{\pi \mu a^2}{c K_p} \frac{\partial y}{\partial t} \]

c--solid fraction

Characteristic Times:

\[ T_1^* = 0.0044 \frac{\pi \mu a^2}{c K_p} \frac{L^4}{EI} \]  (short time)

\[ T_2^* = 0.0789 \frac{\pi \mu a^2}{c K_p} \frac{L^4}{EI} \]  (long time)

Two time constants found by series solution to beam equation

Predicted \( EI \) Vink’s Experiment: \( EI = 700 \ pN \cdot nm^2 \)

Measured \( EI \): \( EI = 17 \times 10^3 \ pN \cdot nm^2 \)  actin (Satcher and Dewey, 1996)
Deflection of Core Protein

distance along fiber (nm)

lateral deflection (nm)

L=400 nm

10 dyn/cm²
Force Amplification

Optical trap: 0.1~0.5 pN (transform receptor)
Drag core protein: $1.4 \times 10^{-3}$ pN
Drag 27 fiber bush: $3.8 \times 10^{-2}$ pN
Vertical shear force actin filament: 0.09 pN
Results: Uniform Laminar flow region

F-actin

Control
DMEM

τ = 10dyn/cm² for 5 h with DMEM,

DMEM + 1% BSA

DMEM + 10% FBS

τ = 10dyn/cm² for 5 h with DMEM,
F-actin is redistributed
(more stress fibers throughout cell)

Control (no flow)

τ = 10dyn/cm² for 5 h with DMEM
τ = 10dyn/cm² for 5 h with DMEM + 1% BSA
τ = 10dyn/cm² for 5 h with DMEM + 10% FBS
Buckling of Initially Curved Beam

Revised Beam Equation

\[ EI \cdot \frac{d^2}{dx^2} \left( y - y_0 \right) = P \left( \delta - y \right) \]

\[ y_0(x) = \arcsin \left( \frac{y}{\delta_0} \right) / a \]

\[ \delta_0 = 0 \text{ nm} \]

\[ P_f = 160 \text{ dyn/cm}^2 \]
ESL Drainage Due to RBC Arrest

Drainage Time

\[ t = \frac{\mu L^2}{12P_c} \int_{L_f}^{L_f_0} \frac{-dL_f}{K_p L_f} \]

Variable K

\[ K_p = \frac{2}{9} \cdot \frac{r^2 L_f / c_0 L_{f_0}}{\sum_{s=0}^{30} q_s \left[ \left( c_0 L_{f_0} / c_{max} L_f \right)^{1/3} \right]^s} \]

Red blood cell membrane

Endothelial cell membrane

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ESL Drainage

![Graph showing ESL Drainage](image)

- Maximum compression
- Constant $K_p$
- Variable $K_p$
- $P_c = 2420 \text{ dyn/cm}^2$

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Dynamic Compression with Goose Down

\[ K_{p0} = 5.0 \times 10^{-10} \text{ m}^2 \]

\[ K_{p0} = 2.0 \times 10^{-9} \text{ m}^2 \]

\[ K_{p0} = 8.0 \times 10^{-9} \text{ m}^2 \]

\[ K_{p0} = 1.6 \times 10^{-8} \text{ m}^2 \]

Experimental data
Feasibility of Supporting a Train Car

<table>
<thead>
<tr>
<th></th>
<th>DYNAMIC COMPRESSION WITH GOOSE DOWN (CASE 1)</th>
<th>ENHANCED LIFT TRAIN TRACK MODEL (L=25m, W =2m) (CASE 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darcy permeability $K_p$</td>
<td>$1.6 \times 10^{-8}$ m$^2$</td>
<td>$1.6 \times 10^{-8}$ m$^2$</td>
</tr>
<tr>
<td>Characteristic time $t_c$</td>
<td>0.1s</td>
<td>3s($v=8.3$ m/s)</td>
</tr>
<tr>
<td>Characteristic length $L_c$</td>
<td>0.40m</td>
<td>25m</td>
</tr>
<tr>
<td>Pressure $P_c$</td>
<td>400Pa</td>
<td>$P_{c2}$</td>
</tr>
</tbody>
</table>

\[
\frac{P_{c2}}{P_{c1}} = \left( \frac{L_2}{L_1} \right)^2 \frac{t_{c1}}{t_{c2}} \frac{K_{p_1}}{K_{p_2}} \quad \Rightarrow \quad P_{c2} = 5.2 \times 10^4 \text{Pa} \quad \text{Lift force} = 260 \text{ tons}
\]
Sketch of the New Train Model in Transverse Plane

- Train car
- Wheels raised
- Sidewall
- Track base
- Thin highly flexible smooth porous plate
- Porous media
- Flexible impermeable membrane
Performance of the Enhanced Lift 50 Ton Train Car

\[ h_2 = 10 \text{ cm} \]

\[ K_{p0} = 5.0 \times 10^{-9} \text{ m}^2 \]

\[ K_{p0} = 1.0 \times 10^{-8} \text{ m}^2 \]

\[ K_{p0} = 1.6 \times 10^{-8} \text{ m}^2 \]
Conclusions

- There is a remarkable dynamic similarity between a red cell gliding on the endothelial glycocalyx and a human skiing though they differ in size by $O(10^{15})$.
- For a given planform without lateral leakage lift increases as the square of the Brinkman permeability parameter $\alpha = h/\kappa^{1/2}$
- For two-dimensional planforms with lateral leakage the lift decreases as $(W/L)^2$.
- The endothelial glycocalyx is an extraordinary structure whose fibers are stiff enough to transmit fluid shear stress to the actin cytoskeleton in initiating intracellular signaling. However, they would easily buckle during red cell arrest were it not for the fluid draining pressure which carries most of the normal load.
- The small elastic restoring force of the fibers allows for a huge reduction in the sliding friction due to the solid phase.
- A highly compressible track with the mechanical properties of goose down is capable of supporting a 50 ton train car traveling at even relatively low speeds with minimal sliding friction. At high speeds there would be little deformation.